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A New Method for Real Time Economic Dispatch Solution Including Wind Farms

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Abstract
Economic dispatch (ED) in the presence of wind farms is of high interest in power system operational planning. Due to the uncertainty in wind speed and wind power, a probabilistic model is required for application in the ED solution. The weibull probability distribution function is a common tool to model the wind speed probabilistic behavior but the main challenge is the non-linearity of wind power generation with respect to wind speed. This causes complexity in the probabilistic ED, which can lead to numerical and time-consuming solution methods. Therefore, linearization of the wind power curve is in the interest of some methods by simply connecting the first point to the end point of power curve by a straight line.
In this work, by developing a conventional objective function for an ED solution, two main contributions are made to obtain a suitable method for fast and also good accuracy results in real time purposes. At first, an improved method is introduced for linearization of wind power curve with respect to the previous works, which increases the performance of modelling with respect to the non-linear model as the base model. The second contribution is to develop an analytical routine for ED by an acceptable time-consuming calculation suitable for real time purposes. The effectiveness of the new approach is tested on two test systems. The results obtained show an improvement in the relative error in ED cost with respect to a real non-linear curve model that reduces an error of about one-fifth regarding the conventional linearization model.

Keywords: *Wind power, Economic dispatch, Real time simulation, Weibull probability function, Linearization.*

1. Introduction
With the rapid growth of wind power penetration all over the world and the increase in the participation of wind power in system generation, the development of economic dispatch (ED) solution methods incorporating wind power is of high interest. In contrast to the conventional generation units, the wind power output is not deterministic and goes up and down randomly. According to this, uncertainty in wind power generation is the main challenge of generation scheduling. Thus it requires a reliable method to overcome the uncertainty in generation allocation to thermal units. One of the most conventional methods for uncertainty modelling of wind power is the probabilistic wind power modelling. As wind power is related to wind speed, and wind speed has a probabilistic behavior, by modelling the wind power with respect to wind speed, and on the other hand, getting a suitable probability distribution function for wind speed, the probability function of wind power can be derived. Some of the famous probability density functions for wind speed probability behavior modeling are as lognormal, gamma, weibull, rayleigh, and inverse gaussian distribution functions. The distribution function of each one is shown in table 1 [1]. Also the details of the specified parameters are described in [1].

Table 1. Wind speed probability distribution functions.

Distribution function	f(v)
Weibull	$\left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{(k-1)} exp\left[-\left(\frac{v}{c}\right)^{k}\right]$
Rayleigh	$\left(\frac{2v}{c^2}\right) exp\left[-\left(\frac{v}{c}\right)^{2}\right]$
Gamma	$\left(\frac{v^{\eta-1}}{\beta^{\eta}\Gamma(\eta)}\right) \exp\left[-\frac{v}{\beta}\right]$
Lognormal	$\left(\frac{1}{v\beta\sqrt{2\pi}}\right) exp\left[-\frac{1}{2}((\frac{\ln(v)-\alpha}{\beta})^2)\right]$
Inverse Gaussian	$\left(\frac{\beta}{2\pi v^3}\right) exp\left[-\frac{\beta(v-\alpha)^2}{2v\alpha^2}\right]$

Nowadays the weibull distribution function is a well-known and widely used one for this propose. The weibull probability density function (PDF) and cumulative distribution function are as (1) and (2), respectively:

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{(k-1)} exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

$$F(v) = 1 - exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (2)$$

where v is the wind speed, and c and k are the scale factor and shape factor, respectively [1].

On the other hand, Wind Energy Conversion (WEC) in wind turbines has a non-linear behavior. A typical wind power-wind speed curve for a 2 MW wind turbine is shown in figure 1.

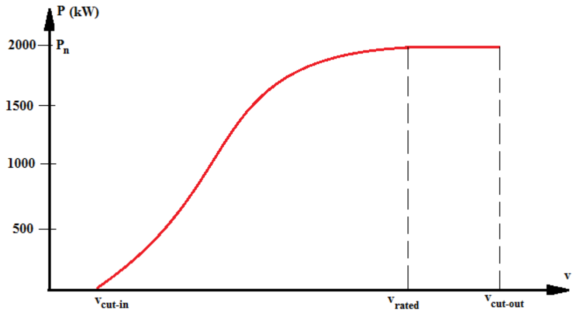

Figure 1. Non-linear wind power curve.

Three main wind speed points related to the wind turbine power curve are defined as v_{cut-in}, v_{rated}, and $v_{cut-out}$. As it has been shown, the wind power is zero for a wind speed less than v_{cut-in}. It has a non-linear curve in the [v_{cut-in}, v_{rated}] interval, where the nominal power (P_n) is generated at v_{rated}. Finally, wind power is constant in [v_{rated}, $v_{cut-out}$] and it falls down to zero at $v_{cut-out}$.

The wind power curve has been reviewed in some articles in details [2-5]. Some non-linear formulations for the wind turbine power curve are presented in [2]. The famous non-linear model for this curve is the third-order polynomial. The main formulation can be found in the literature, as shown in (3) [2, 6].

$$P_w = \begin{cases} 0 & v < v_{cut-in} \text{ , } v > v_{cut-out} \\ P_n\left(\dfrac{v^3 - v_{cut-in}^3}{v_{rated}^3 - v_{cut-in}^3}\right) & v_{cut-in} \le v \le v_{rated} \\ P_n & v_{rated} \le v \le v_{cut-out} \end{cases} \quad (3)$$

where P_w and P_n are wind power at wind speed v and nominal (rated) wind power, respectively. This formulation has been shown with more simplification in some other literatures by ignoring v_{cut-in} in a second relation both in the numerator and denominator as (4).

$$P_w = \begin{cases} 0 & v < v_{cut-in} \text{ , } v > v_{cut-out} \\ P_n \dfrac{v^3}{v_{rated}^3} & v_{cut-in} \le v \le v_{rated} \\ P_n & v_{rated} \le v \le v_{cut-out} \end{cases} \quad (4)$$

The main challenge to get the wind power probability distribution function is the non-linear relation between the wind speed and the wind power, which makes it difficult to convert the wind speed probability function to the wind power probability function. In this way, a simple but approximate approach is to linearize wind speed–wind power relation in the [v_{cut-in}, v_{rated}] interval. This approach has been used in some articles [7-9]. However, the linearization method used in these articles is so simple by connecting the starting point to the ending point by a straight line in this interval. However, this approach has an unacceptable error with respect to the real non-linear power curve, as it will be shown later. Then one of the main contributions of this paper is to develop a linearization method to yield an acceptable error with respect to the real model.

Another base of the economic dispatch solution in the presence of wind farms is to develop a suitable objective function. As we know, the main section of the thermal unit economic dispatch objective function is the generation operation cost usually defined as a second-order function with respect to power generation. Another section of this objective function is the cost function of wind power. Some terms may be added to this objective function to show the costs that are related to the uncertainty in wind power.

In [7], a basic approach for the economic dispatch model has been introduced in the presence of wind farm. The objective function is formed by the two sections previously mentioned, in addition to a term that shows the cost of the required reserve to overcome wind power uncertainty, and also another term that shows the penalty for not using all the available wind power. The weibull probability distribution function and also wind power–wind speed linearization have been suggested in the mentioned paper. However, it has not focused on the optimization method. In [8], the same approach has been used for the objective function and linearization method, and optimization has been carried out using the Particle Swarm Optimization (PSO). In [9], another optimization algorithm named the firefly algorithm has been used including network power loss by B-coefficient transmission loss calculation. Some other papers related to the ED-incorporated wind farm have been addressed in [10-21].

As it can be seen, all the optimization methods are based on the evolutionary algorithms. In this paper, based on [7] for ED objective function development and proposed wind power linearization, two main contributions are made. At first, a new linearization approach is used to get a better fitness for the non-linear real curve of wind power. This improves the accuracy much better and goes toward the real optimum point. The second contribution is to present an analytical method for the probability function integration used for objective function calculation that causes the ED optimization solution to be analytical and of less computation time. This is a good advantage of the

proposed method for real time use. The usefulness of the two approaches is shown in tow test systems, and the results are obtained by the same approach derived in [7], which shows a considerable improvement in the results.
In the following, the wind power model is described in part 2. Part 3 describes some bases for the mathematical calculations used in the objective function optimization. The optimization procedure is presented in part 4. Part 5 shows the simulation and the analysis result, and part 6 describes the conclusion.

2. Model description

The main formulation for wind power generation is as (5) (similar to (4)):

$$P_w = \begin{cases} 0 & v<v_{cut\text{-}in}\ ,\ v>v_{cut\text{-}out} \\ k_w v^3 & v_{cut\text{-}in} \le v \le v_{rated} \\ P_n & v_{rated} \le v \le v_{cut\text{-}out} \end{cases} \quad (5)$$

where k_w is defined as [2]:

$$k_w = P_n / v_{rated}^3 = 0.5 n_t\, C_p \eta\, A\rho \quad (6)$$

In the above equation, n_t, C_p, η, A, and ρ are the number of wind turbines in a wind farm, the power coefficient for wind turbine, the wind turbine-generator efficiency, the turbine area, and the air density.
As the wind speed is probabilistic, it requires to use a suitable Probability Density Function (PDF). Nowadays, the wiebull probability function is a completely conventional function for this purpose. The PDF and CDF of this probability function are illustrated in (1) and (2), respectively.
In this way, the cumulative probability of wind power can be written as:

$$P_r(P_w=0)=F(v_{cut\text{-}in})+(1\text{-}F(v_{cut-out}))= 1\text{-}\exp\left[-\left(\frac{v_{cut\text{-}in}}{c}\right)^k\right]+\exp\left[-\left(\frac{v_{cut\text{-}out}}{c}\right)^k\right] \quad (7)$$

$$P_r(0<P_w<P_n)=F(v_{rated})\text{-}F(v_{cut\text{-}in})= \exp\left[-\left(\frac{v_{cut\text{-}in}}{c}\right)^k\right]-\exp\left[-\left(\frac{v_{rated}}{c}\right)^k\right] \quad (8)$$

$$P_r(P_w=P_n)=F(v_{cut\text{-}out})\text{-}F(v_{rated})= \exp\left[-\left(\frac{v_{rated}}{c}\right)^k\right]-\exp\left[-\left(\frac{v_{cut\text{-}out}}{c}\right)^k\right] \quad (9)$$

Now, the economic dispatch objective function is described. The conventional form of this function incorporating wind farm can be written as [7]:

$$\min Cost=\min\Big[\sum_{i=1}^{N_g} C_i(P_i)+\sum_{i=1}^{N_w} C_{wi}(P_{wi})+ \sum_{i=1}^{N_w} CP_{wi}(W_{wi}\text{-}P_{wi})+ \sum_{i=1}^{N_w} CR_{wi}(P_{wi}\text{-}W_{wi})\Big] \quad (10)$$

In (10), Cost, C, C_w, CP_w, and CR_w stand for the total operation cost, the thermal unit generation cost, the wind power cost, the penalty cost function for unused available wind power, and the required reserve cost function, respectively. Also P, P_w, and W_w show the thermal unit generation and real and expected wind power, respectively. Also N_g and N_w are the generation unit number and the wind farm number, respectively. Now, each one of the four terms of this objective function is explained.
The first term is the thermal unit operation cost as:

$$\sum_{i=1}^{N_g} C_i(P_i) = \sum_{i=1}^{N_g} a_i P_i^2 + b_i P_i + c_i \quad (11)$$

where a, b, and c are the cost function coefficients.
The second term defines the wind power operation cost directly related to wind power:

$$\sum_{i=1}^{N_w} C_{wi}(P_{wi}) = \sum_{i=1}^{N_w} d_i P_{wi} \quad (12)$$

In (12), d stands for cost per one unit power. The last two functions in (10) model the penalty cost function for unused available wind power, and the required reserve cost function are calculated as:

$$\sum_{i=1}^{N_w} CP_{wi}(W_{wi}-P_{wi}) = \sum_{i=1}^{N_w} K_{pi}(W_{wi}-P_{wi}) \quad (13)$$

$$\sum_{i=1}^{N_w} CR_{wi}(P_{wi}-W_{wi}) = \sum_{i=1}^{N_w} K_{Ri}(P_{wi}-W_{wi}) \quad (14)$$

K_p and K_R are the corresponding coefficients. It is clear that (W_w-P_w) and (P_w-W_w) are the probabilistic variables because of W_w as a probabilistic variable, and are defined as:

$$(W_{wi}-P_{wi}) = \int_{P_{wi}}^{P_{ni}} (W_{wi}-P_{wi}) f(W_{wi}) d(W_{wi}) \quad (15)$$

$$(P_{wi}-W_{wi}) = \int_{0}^{P_{wi}} (P_{wi}-W_{wi}) f(W_{wi}) d(W_{wi}) \quad (16)$$

Finally, the main constraints of the mentioned objective function are:

$$P_i^{min} \le P_i \le P_i^{max} \quad (17)$$

$$0 \le P_{wi} \le P_{ni} \quad (18)$$

$$\sum_{i=1}^{N_g} P_i + \sum_{i=1}^{N_w} P_{wi} = D \quad (19)$$

P^{min} and P^{max} are the minimum and maximum limits of unit generation, respectively, and D stands for the demand. The objective function (10) should be optimized with respect to P_i's and P_{wi}'s. However, the main complexity in this method is the calculation of (15) and (16), where the relation between W_w and v is non-linear, modelled by (5).

As said earlier, the well-known linearization procedure for (5) is as (20) [7]:

$$P_w = \begin{cases} 0 & v < v_{cut-in}, v > v_{cut-out} \\ P_n \dfrac{(v - v_{cut-in})}{(v_{rated} - v_{cut-in})} & v_{cut-in} \le v \le v_{rated} \\ P_n & v_{rated} \le v \le v_{cut-out} \end{cases} \quad (20)$$

In other words, the point (v_{cutin}, 0) is connected to point (v_{rated}, P_n) by a straight line.

In this work, a new linearization method is developed as a Least Square Error (LSE) line fitting between the two points mentioned. As the LSE linearization is a well-known method for curve fitting to get the best fitness, this paper suggests this method instead of simply linearization, as said before. Verification of the proposed method will be done in part 5 by simulation. The desired linearization routine is described here. Eq. (21) is similar to (20) but m and n are calculated by the line fitting calculation:

$$P_w = \begin{cases} 0 & v < v_{cut-in}, v > v_{cut-out} \\ mv + n & v_{cut-in} \le v \le v_{rated} \\ P_n & v_{rated} \le v \le v_{cut-out} \end{cases} \quad (21)$$

The LSE function can be shown as:

$$LSE = \int_{v_{cut-in}}^{v_{rated}} [k_w v^3 - (mv + n)]^2 \quad (22)$$

Finally, the necessary equations for determination of the coefficient m and n can be written in a matrix form as:

$$\begin{bmatrix} (v_{rated}^3 - v_{cut-in}^3)/3 & (v_{rated}^2 - v_{cut-in}^2)/2 \\ (v_{rated}^2 - v_{cut-in}^2)/2 & (v_{rated} - v_{cut-in}) \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} k_w(v_{rated}^5 - v_{cut-in}^5)/5 \\ k_w(v_{rated}^4 - v_{cut-in}^4)/4 \end{bmatrix} \quad (23)$$

Figure 2 shows a typical non-linear P-v curve and also two methods for P-v linearization.

It should be noted that in the line fitting method, the start and stop points ((v_{cut-in}, 0), (v_{rated}, P_n)) of the non-linear curve do not necessarily stay on the line fitted. Thus by an approximation, a small part of the fitted line less than v_{cut-in} and more than v_{rated} is ignored.

3. Mathematical calculations

In this section, the mathematical calculations of (15) and (16) are presented for the wind power linear model. It should be mentioned that the optimizations are down in the interval [v_{cut-in}, v_{rated}]. Thus the next equations are established:

$$W_w = mv + n \quad (24)$$

$$d(W_w) = mdv \quad (25)$$

$$V_w = \frac{P_W - n}{m} \quad (26)$$

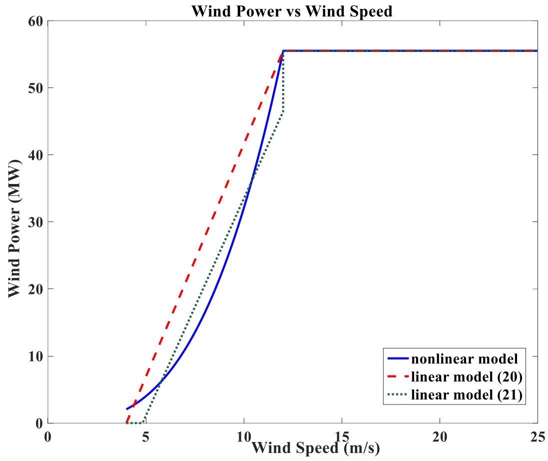

Figure 2. Non-linear and linearized models.

At first, the procedure for calculation of (15) is determined. For simplicity, only one wind farm is assumed. Thus the index i is ignored. Now, (15) can be converted as:

$$\int_{P_w}^{P_n} (W_w - P_w) f(W_w) d(W_w)$$
$$(P_n - P_w)(\exp\left[-\left(\frac{v_{rated}}{c}\right)^k\right] - \exp\left[-\left(\frac{v_{cut-out}}{c}\right)^k\right]) \quad (27)$$
$$+m \int_{V_w}^{V_{rated}} (mv + n - P_w) f(v) d(v)$$

The first term on the right side (non-integral part) shows the area that corresponds to the interval [v_{rated}, $v_{cut-out}$], where the wind power is fixed to P_n. The second term on the right side of (27) can be converted to:

$$m \int_{V_w}^{V_{rated}} (mv+n-P_w) f(v) d(v) = m(n-P_w) \int_{V_w}^{V_{rated}} f(v) d(v)$$
$$+m^2 \int_{V_w}^{V_{rated}} v f(v) d(v) \quad (28)$$

Also the second part of the right side (28) should be done by a part-by-part integration. Thus we have:

$$m \int_{V_w}^{V_{rated}} (mv+n-P_w) f(v) d(v) = m(n-P_w)(F(v_{rated})$$
$$-F(V_w)) + m^2[(v_{rated} F(v_{rated}) - V_w F(V_w)) - \int_{V_w}^{V_{rated}} F(v) d(v)] \quad (29)$$

In order to calculate the integral part of the right side of (29), the shape factor of the weibull function (k) should be known. Here, this parameter is assumed 1 (k = 1). In other shape factors, the entire procedure is the same, except for the mentioned integral. Thus the last part of (29) is written as:

$$\int_{V_w}^{V_{rated}} F(v)d(v) = (v_{rated} - V_w) - c(\exp\left[-\left(\frac{V_w}{c}\right)\right] - \exp\left[-\left(\frac{v_{rated}}{c}\right)\right]) \tag{30}$$

Finally, (27) can be summarized as:

$$\int_{P_w}^{P_n} (W_w - P_w) f(W_w) d(W_w) = S1 + M1 - N1 + R1 \tag{31}$$

where S1, M1, N1, and R1 are as below:

$$S1 = \left(P_n - (mV_w + n)\right) (\exp\left[-\left(\frac{v_{rated}}{c}\right)\right] - \exp\left[-\left(\frac{v_{cut-out}}{c}\right)\right]) \tag{32}$$

$$M1 = m^2(v_{rated}\left(1 - \exp\left[-\left(\frac{v_{rated}}{c}\right)\right]\right) - V_w(1 - \exp\left[-\left(\frac{V_w}{c}\right)\right])) \tag{33}$$

$$N1 = m^2((v_{rated} - V_w) - c\left(\exp\left[-\left(\frac{V_w}{c}\right)\right]\right) - \exp\left[-\left(\frac{v_{rated}}{c}\right)\right]) \tag{34}$$

$$R1 = -m^2 V_w \left(\exp\left[-\left(\frac{V_w}{c}\right)\right] - \exp\left[-\left(\frac{v_{rated}}{c}\right)\right]\right) \tag{35}$$

Now, the same procedure is done to calculate (16). At first, (16) is converted to:

$$\int_0^{P_w} (P_w - W_w) f(W_w) d(W_w) = P_w \left(1 - \exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right] + \exp\left[-\left(\frac{v_{cut-out}}{c}\right)\right]\right) + m\int_{v_{cut-in}}^{V_w} (P_w - mv - n) f(v) d(v) \tag{36}$$

Again, the first part of the right side shows the area corresponding to zero wind power (less than v_{cut-in} or more than $v_{cut-out}$). Calculation of the integral part of (36) is similar to the previous one. Thus the final result is shown as:

$$\int_0^{P_w} (P_w - W_w) f(W_w) d(W_w) = S2 + M2 - N2 + R2 \tag{37}$$

where S_2, M_2, N_2, and R_2 are as below:

$$S2 = (mV_w + n)(1 - \exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right] + \exp\left[-\left(\frac{v_{cut-out}}{c}\right)\right]) \tag{38}$$

$$M2 = m^2 V_w \left(\exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right] - \exp\left[-\left(\frac{V_w}{c}\right)\right]\right) \tag{39}$$

$$N2 = m^2[V_w\left(1 - \exp\left[-\left(\frac{V_w}{c}\right)\right]\right) - v_{cut-in}\left(1 - \exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right]\right)] \tag{40}$$

$$R2 = m^2[(V_w - v_{cut-in}) - c\left(\exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right] - \exp\left[-\left(\frac{V_w}{c}\right)\right]\right)] \tag{41}$$

For completion, for the main procedure to get the final form of the objective function, (31) and (37) should be substituted in (15) and (16), respectively, and then the modified form of the objective function (10) is derived.

4. Optimization Procedure

Another contribution of this paper is to suggest an analytical method for ED solution in the presence of wind farms that is less time-consuming and suitable for real time purposes, which is described below.

A suitable method for minimization of (10) is the Lagrange Multiplier method as (42).

$$\text{minLG} = \min[\sum_{i=1}^{N_g} C_i(P_i) + \sum_{i=1}^{N_w} C_{wi}(P_{wi}) + \sum_{i=1}^{N_w} CP_{wi}(W_{wi} - P_{wi}) + \sum_{i=1}^{N_w} CR_{wi}(P_{wi} - W_{wi}) - \lambda(\sum_{i=1}^{N_g} P_i + \sum_{i=1}^{N_w} P_{wi} - D)] \tag{42}$$

Here, only one wind farm and also the shape factor equal to 1 are assumed. Thus we have:

$$minLG = \min[\sum_{i=1}^{N_g} [a_i P_i^2 + b_i P_i + c_i] + d(mV_w + n) + K_p(S1 + M1 - N1 + R1) + K_R(S2 + M2 - N2 + R2) - \lambda(\sum_{i=1}^{N_g} P_i + mV_w + n - D)] \tag{43}$$

Now, the partial derivatives of LG with respect to P_i's, V_w, and λ are derived and put equal to zero.

$$\frac{\partial LG}{\partial P_i} = 2a_i P_i + b_i - \lambda = 0 \tag{44}$$

$$\begin{aligned}\frac{\partial LG}{\partial V_w} = {} & m(d - \lambda) + K_p\Big\{-m\left(\exp\left[-\left(\frac{v_{rated}}{c}\right)\right] - \exp\left[-\left(\frac{v_{cut-out}}{c}\right)\right]\right) - m^2\left[\left(1 - \exp\left[-\left(\frac{V_w}{c}\right)\right]\right) + \left(\frac{V_w}{c}\right)\exp\left[-\left(\frac{V_w}{c}\right)\right]\right] + m^2(1 - \exp\left[-\left(\frac{V_w}{c}\right)\right] \\ & - m^2\left(\left(\exp\left[-\left(\frac{V_w}{c}\right)\right] - \exp\left[-\left(\frac{v_{rated}}{c}\right)\right] - \frac{V_w}{c}\exp\left[-\left(\frac{V_w}{c}\right)\right]\right)\right)\Big\} \\ & + K_R\Big\{m\left(1 - \exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right] + \exp\left[-\left(\frac{v_{cut-out}}{c}\right)\right]\right) + m^2\left[\exp\left[-\left(\frac{V_{cut-in}}{c}\right)\right] - \exp\left[-\left(\frac{V_w}{c}\right)\right] + \left(\frac{V_w}{c}\right)\exp\left[-\left(\frac{V_w}{c}\right)\right]\right] - m^2(1 \\ & - \exp\left[-\left(\frac{V_w}{c}\right)\right] + \frac{V_w}{c}\exp[-\left(\frac{V_w}{c}\right)]) + m^2\left(1 - \exp\left[-\left(\frac{V_w}{c}\right)\right]\right)\Big\} = 0\end{aligned} \tag{45}$$

$$\frac{\partial LG}{\partial \lambda} = \sum_{i=1}^{N_g} P_i + mV_w + n - D = 0 \tag{46}$$

By solving the above three equations simultaneously, the variables P_i's and V_w are found.

For this purpose, a simple method is described here. From (44), all P_i's can be calculated by λ. Then the sum of P_i's are calculated as:

$$\sum_{i=1}^{N_g} P_i = 0.5(s_2\lambda - s_1) \tag{47}$$

where s_1 and s_2 are:

$$s_1 = \sum_{i=1}^{N_g} \frac{b_i}{a_i} \tag{48}$$

$$s_2 = \sum_{i=1}^{N_g} \frac{1}{a_i} \tag{49}$$

By substitution of (47) into (46), we have:

$$\lambda = MV_w + N \tag{50}$$

where M and N are as below:

$$M = \frac{-2m}{s_2} \tag{51}$$

$$N = \frac{2(D-n)+s_1}{s_2} \tag{52}$$

On the other hand, by simplifying (45), the variable V_w appears as the term of exp($-V_w/c$) only. Thus if we assume this term as Z, we have a very simple equation as:

$$AZ + B + m\lambda = 0 \tag{53}$$

where A and B are defined as:

$$A = (K_p + K_R)m^2 \tag{54}$$

$$\begin{aligned} B = K_p m\left[\exp\left[-\left(\frac{v_{rated}}{c}\right)\right] - \exp\left[-\left(\frac{v_{cut-out}}{c}\right)\right] \right. \\ \left. - m\exp\left[-\left(\frac{v_{rated}}{c}\right)\right]\right] \\ +K_R m\left[-1 + \exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right] - \exp\left[-\left(\frac{v_{cut-out}}{c}\right)\right] \right. \\ \left. - m\exp\left[-\left(\frac{v_{cut-in}}{c}\right)\right]\right] - md \end{aligned} \tag{55}$$

Now, a simple iteration method can be used to solve (50) and (53) simultaneously. At first, with an initial guess of V_w, (50) yields the initial guess for λ. Then from (53), Z is found and thus the corrected value of V_w is calculated. Again, the corrected value of V_w is used to obtain the new value of λ from (50) and also calculate the new value of Z from (53). This cycle is repeated until convergence is achieved. Generally, a few iterations are required to get the results with an acceptable error.

5. Simulation

In this section, the thermal unit incorporated wind power economic dispatch solution is considered in two test systems as four/six thermal unit generation systems. In the first system, different cost coefficient scenarios are considered to have more details in analysis. The second system is used for a better verification of the proposed model and a better comparison with the previous model.

5.1. Four unit test system

The cost functions of the first system generation units are as [22]:

$C_1=0.01Pg_1^2+1.8Pg_1+300$ (\$)
$C_2=0.012Pg_2^2+2.24Pg_2+210$ (\$)
$C_3=0.006Pg_3^2+2.35Pg_3+290$ (\$)
$C_4=0.008Pg_4^2+2.5Pg_4+340$ (\$)

The demand is considered as 600 MW. No up or down generation unit constraint is considered. Also a wind farm is added by the following data:

A = 4000 m^2, ρ = 1.255 Kg/m^3, Cp = 0.4, η = 0.8, n_t = 40.

The v-P wind power parameters and weibull probability function parameters are:

v_{cut-in} = 4 m/s, v_{rated} = 12 m/s, $v_{cut-out}$ = 25 m/s, c = 8, k = 1

Now, three scenarios are defined for different case studies. The first one is defined by ignoring the wind power incorporation to get the base case result. Scenario 2 describes the case study with wind power incorporation and v-P non-linear model (real case). Finally, scenario 3 is similar to the previous scenario but with two linear models described before. By comparison of scenarios 2 and 3, the proposed linear model is verified.

5.1.1. Scenario 1

As said earlier, at first, a preliminary analysis is done by ignoring the wind farm. The result for the thermal unit economic dispatch is:

Pg_1=149.0351 MW Pg_2=105.8626 MW
Pg_3=202.5585 MW Pg_4=142.5439 MW

The cost is 3243 (\$).

5.1.2. Scenario 2

In this scenario, the wind farm is considered by cost coefficients as:

d = 1 (\$/MW) K_P = 1 (\$/MW) K_R = 2 (\$/MW)

Also the non-linear model for wind power is considered. Thus it requires the numerical integration of (15) and (16). As a conventional method, the Euler numerical integration is used. The point-by-point calculation of the objective function gives the cost curve *vs.* wind power, shown in figure 3.

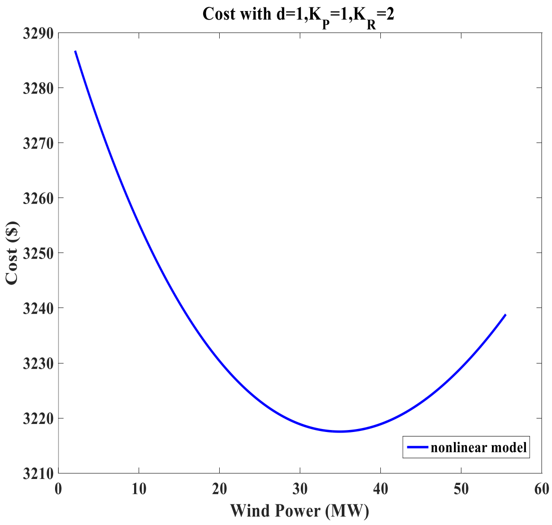

Figure 3. Non-linear model total cost function.

The minimum point of cost curve can be extracted as 3217 ($) and wind power as 34.94 MW generated by 10.28 m/s wind speed. This is considered as the base to comprise the two linearization models described before.

5.1.3. Scenario 3

This scenario is similar to scenario 2 but with two linear models of wind power described by (20) and (21). In the first step, the cost function (10) is calculated for each one of the two models by the point-by-point calculation, similar to the non-linear model. The cost curves are shown in figure 4. The blue curve is the non-linear model, the red curve is the linear model (20), and the green curve is the linear model (21) (proposed model).

It is obvious that the model described by (20) has a high deviation from the real model, especially at the minimum point, and the deviation increases by an increase in the wind power. However, the proposed model has a much less error and follows the real model by a good fitness.

The minimum point for each linear model is as 3226 ($) and wind power as 31.27 MW generated by 8.51 m/s wind speed for the first model and 3219 ($) and wind power as 34.02 MW generated by 10.08 m/s wind speed for the second model. It is again clear that the desirability of the proposed model result with respect to the other model.

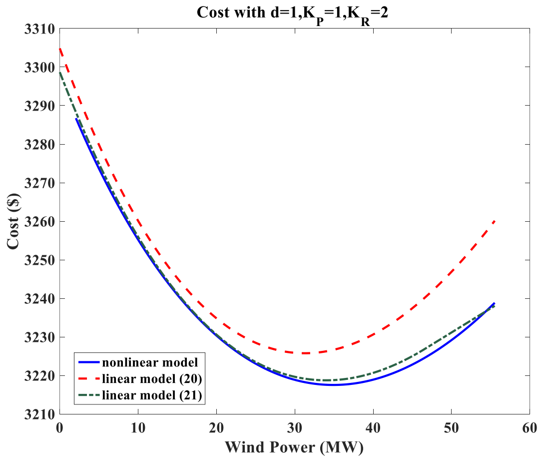

Figure 4. Non-linear and linear models total cost functions.

5.1.4. Optimization calculation

Now, the developed method is ready for use in order to find the minimum point of two linear model cost functions by the analytical procedure.

As mentioned earlier, an initial guess for V_w is initially made. Suppose that V_w equals 12 m/s as the initial guess. By a simple iterative solution described before to obtain the corrected value of V_w, the results are found for two linear models. Thus the minimum point for the first linear model is as 3230 ($) for the cost function and 32.60 MW for the wind power generated at 8.70 m/s wind speed. Similar results for the second model are 3221 ($) for the cost function and 32.27 MW for the wind power generated at 9.81 m/s wind speed. The iteration for each one is only six, which shows a good convergence in the solution and also a low calculation time. This shows an adequate performance of the proposed solution routine for use in real time purposes.

As it can be seen, the results derived by point-by-point calculation and analytical routine are very close together.

5.1.5. Sensitivity analysis

In this section, a sensitivity analysis is done to get a clear view of the economic dispatch in the presence of wind power uncertainty and also to show the adequate performance of the proposed approach for wind power curve linearization and the analytical ED solution routine. This is done by changing the three parameters d, K_P, and K_R. At first, it is assumed that no penalty exists for unused available wind power ($K_P = 0$). Table 2 shows the results for total cost, allocated wind power, and the corresponding wind speed for some values of d and K_R. It is understandable that by increasing K_R, the allocated wind power decreases. Also increasing the wind power operation cost causes less allocated wind power.

Table 2. Sensitivity results by $K_P = 0$.

d	K_P	K_R	Cost($)	Power (MW)	V(m/s)
1	0	1.5	3190	40.15	11.02
1	0	1.75	3202	30.83	9.59
1	0	2	3211	24.65	8.63
1	0	2.25	3218	20.23	7.95
1	0	2.5	3223	16.91	7.44
2	0	1.5	3221	23.47	8.45
2	0	1.75	3227	18.17	7.63
2	0	2	3231	14.46	7.06
2	0	2.25	3234	11.71	6.63
2	0	2.5	3236	9.60	6.31

Now, the same analysis is done by $K_P = 2$. Again, table 3 shows the same results as table 2 in this analysis.

Table 3. Sensitivity results by $K_P = 2$.

d	KP	KR	Cost ($)	Power (MW)	V (m/s)
1	2	1.5	3191	46.97	12.08
1	2	1.75	3209	41.23	11.19
1	2	2	3224	36.65	10.48
1	2	2.25	3236	32.90	9.91
1	2	2.5	3247	29.76	9.42
2	2	1.5	3233	37.87	10.67
2	2	1.75	3246	33.48	10.00
2	2	2	3257	29.91	9.44
2	2	2.25	3266	26.93	8.93
2	2	2.5	3274	24.41	8.59

It is clear that by including penalty for the unused available wind power, the economic dispatch solution goes toward more wind power allocation.

5.2. Six unit test system

For a better comparison of the proposed model, IEEE-30 bus/six unit test system is also simulated. The cost functions of generation units are as [23]:

$$C_1=0.00375Pg_1^2 + 2Pg_1 \quad (\$)$$
$$C_2=0.01750Pg_2^2 + 1.75Pg_2 \quad (\$)$$
$$C_3=0.06250Pg_3^2 + 1.00Pg_3 \quad (\$)$$
$$C_4=0.00834Pg_4^2 + 3.25Pg_4 \quad (\$)$$
$$C_5=0.02500Pg_5^2 + 3.00Pg_5 \quad (\$)$$
$$C_6=0.02500Pg_6^2 + 3.00Pg_6 \quad (\$)$$

The demand is considered as 250 MW. The wind farm parameters are completely similar to the previous test case.

The results for thermal unit economic dispatch without wind farm incorporation are:

$Pg_1 = 172.8310$ MW $\quad Pg_2 = 44.1781$ MW
$Pg_3 = 18.3699$ MW $\quad Pg_4 = 2.7717$ MW
$Pg_5 = 5.9247$ MW $\quad Pg_5 = 5.9247$ MW

The cost is 654.9789 ($).

Now, a similar simulation as scenario 3 for the previous case is done. The cost curves for real and two linearized models are shown in figure 5.

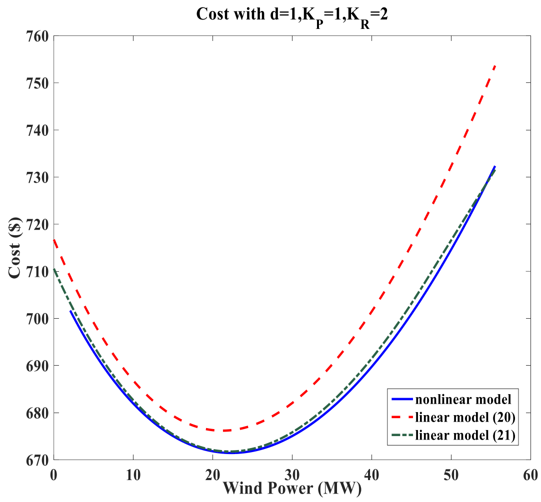

Figure 5. Non-linear and linear modeltotal cost functions

Again the model described by (20) has a high deviation from the real model, especially at the minimum point but the proposed model has a much less error and follows the real model by a good fitness.

The minimum point for the real model is as 671.5 ($) and wind power as 22.34 MW generated by 8.86 m/s wind speed. Also the minimum point for each linear model is as 676.2 ($) and wind power as 21.17 MW generated by 7.05 m/s wind speed for the first model and 671.8 ($) and wind power as 22.10 MW generated by 8.24 m/s wind speed for the second model.

Now the ED optimization is done by the proposed algorithm. Again, by an initial guess for V_w to be equal to 12 m/s, the results are found for two linear models. The minimum point for the first linear model is as 682.3 ($) for the cost function and 22.31 MW for the wind power generated by 7.21 m/s wind speed. Similar results for the second model are 671.8 ($) for the cost function and 20.71 MW for the wind power generated by 8.02 m/s wind speed. The iteration for each one is only six, which shows a good convergence in the solution and also a low calculation time.

6. Conclusion

The uncertainty and variability of wind power generation are two main challenges that require the probabilistic routines both in the planning and the operation time zones. In this way, the real time economic dispatch incorporating wind farm is of high interest in the power system operation.

In this paper, by development of a new method for wind power curve linearization, a good improvement in accuracy with respect to the non-linear real model was achieved. On the other hand, by presenting an analytical routine for ED solving, a fast and reasonable time-consuming algorithm was suggested, which is suitable for a real time simulation.

The simulation results for two different test systems showed an improvement in accuracy in the proposed linearized model, where the deviation in minimum point of the cost function with respect to real model was reduced to about one-fifth in the proposed model with regard to the conventional linearized model. On the other hand, the error in the wind speed result was more than the error in wind power result in the conventional model. It could prove more accuracy for the proposed linear model with respect to the conventional model.

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