

# Comparison of Monte Carlo Simulation and Genetic Algorithm in Optimal Wind Farm Layout Design in Manjil Site based on Jensen Model

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## Abstract

An optimal arrangement of turbines in wind farms is very important in order to achieve maximum energy at the lowest cost. In the present work, the use of Vestas V-47 wind turbine and uniform one-way wind in achieving the optimal arrangement of horizontal axis turbines in Manjil using the genetic and Monte Carlo algorithms is investigated. The Jensen model is used to simulate the wake effect on the downstream turbines. The objective function is considered as the ratio of cost to power of the power plant. The results obtained show that the Monte Carlo method compared to the genetic algorithm will give a better result. Under the same conditions, the Monte Carlo algorithm will give 29% and 40% better results in terms of the number of turbines and output power, respectively. In terms of optimization, in the Monte Carlo algorithm, its fitness value is 16% less than the genetic algorithm, which indicates its better optimization.

**Keywords:** Wind Turbine, Optimization, Monte Carlo Method, Genetic Algorithm, Farm Layout.

## 1. Introduction

The global energy crisis has put the wind energy at the focus of attention as a clean and unlimited energy source for the future. The first attempts regarding the use of wind energy dates back to the industrial revolution era when the wind turbines (WTs) were first used as a means of energy harvesting to generate electricity. WTs are the devices widely used in the agricultural and commercial sectors to derive energy from the free stream wind near the ground, and within a planetary boundary layer. The wind flow characteristics are remarkably affected by the energy extraction process due to retarding the wind speed at downstream. A cluster of windmills installed together on the ground are called a windfarm that perform the electricity extraction efficiently compared to the fossil fuel power stations [1, 2]. The local wind features are benchmarks for a site to be selected as a windfarm. Equally importantly, land geology and accessibility, and environmental concerns are the other key factors. The energy produced by a turbine placed in the downstream of another turbine will deteriorate as a result of energy deficit caused by the upstream turbine [3, 4]. A momentum deficit occurs behind a turbine, and

the flow velocity will be lower compared to the upstream values. Thus the performance of windfarm turbines that are placed in the downstream sections of wakes with lower energy will be reduced considerably [5]. At a distance far enough from behind the turbine, the wind speed will have returned to its free stream value. It is very important to understand such power degradation characteristics and take them into account when designing the turbine placement and WF layout. Obviously, the WF concept will be reasonable and cost-effective on the condition that all factors are considered. Consequently, clustering WTs in a certain number of locations will give the highest wind energy potential. Since different factors play a role in forming the wake flow behind a turbine array, wake size prediction, defining the location and velocity of each turbine, and modeling the interactions between multiple turbines wakes is a sophisticated problem in fluid dynamics, which can be well-handled using computer simulations. It is noteworthy that accurate modeling of atmospheric interactions is almost impossible and approximation methods are widely used to tackle the problem [6–10]. In the recent years, attempts have been made for the

selection of suitable wake models and simulation approaches. As far as the classical mechanics is concerned, the accurate analysis and prediction of fluctuations to the wind flow as the flow encounters an obstacle is very complicated. The most fundamental difficulty associated with this problem is the modeling of atmospheric turbulence. In addition, complex boundary conditions regarding the WT wake makes the problem more complex. The main feature of WT blade rotating motion is the adverse pressure gradients, which can result in a severe alteration of the downstream flow pattern as well as upwind of the pertinent WT. Obviously, a reasonable and efficient windfarm design, cannot be achieved without using both the experimental and analytical techniques. The main focus of this paper is to present an optimal solution to the problem of windfarm design, based on Jensen wake model, employing GA and MCm. The main objective of the WF wake study is to formulate the interactions among a number of large WTs all together form an extended array or "WF" [11, 12].

A specific consideration must be given to defining the amount of power output reduction that down WTs suffer as a result of energy extraction by the upwind units, in case when several WTs are aligned with the wind. The most important parameter is the distance between the units. The Manjil site is a rectangular with an inter-unit spacing of 2500\*2500 m<sup>2</sup>. It is expected that the overall power reduction due to the loss of wakefulness can be obtained for each given wind speed as a function of unit distance. Thus a developed technique for performing a cluster function is required to specify the interaction between the units. A new technique in this research work for estimating the power of a windfarm design is presented, which seems to solve the problems presented by the reduced-order awakening modeling. Obviously, the power reduction is significant, and strongly depends on the wind direction. When planning the WT clusters, this power reduction should be considered so that the annual estimates for different cluster configurations at a particular site can be estimated. When calculating the power reduction for a cluster, several factors must be considered. The power of WTs to generate electricity is evident in technology; especially where there are many wind resources, the wind power converters are more economically competitive than the other conventional power generation units are. However, in the case of large multi-megawatt power plants, a large number of WTs must be used, and the overall efficiency of

the windfarm will be greatly affected by their placement. The present work offers a new approach to the problem of WT placement. The adopted method is based on the principles of evolutionary calculations and random sampling. The main feature of the genetic optimization procedure is the independency of the local optimization or gradients.

On the other hand, the Monte Carlo method (MCm) provides us with the feature of random sampling, which means that traversing the solution space in each step of the algorithm randomly and comparing the solution with the previous best candidate. The effect of WTs on each other and wind variations are two important factors that must be considered in order to achieve the maximum power for the minimum installation cost.

The problem constraints like the WF size, wind distribution, and WT specification must be defined correctly to be able to simulate the energy decrease pattern for a reasonable number of possible configurations in order to evaluate whether it is the optimum solution.

In order to solve this problem, two separate algorithms are proposed, one for evaluating the performance of the windfarm, and the other for the optimization procedure. The wind wake is simulated using the Jensen model, while the optimization technique using GA and MCm. For an extensive utilization of the wind energy, WTs are placed in special arrangements in windmills. There are limited areas with wind resources to place WTs together in clusters or WFs. WTs often intervene with each other on windfarms. The Downstream flow behind the turbine has a low velocity, a strong wind shear, and usually an intense turbulence. Hence another WT located along the wind direction behind the first turbine is probably to generate less energy than the undisturbed one by a value that is decreased with increasing distance. The wind shear and turbulence are known to be the two causes of dynamic loads in WTs. Consequently, with this concept, an accurate description of the flow behind WTs is required to study WFs. The flow characteristics behind several WTs have been extensively studied over the past two decades. The initial case report is by Templin [12].

The theoretical model presented in this report discusses the effect of an infinite WF in terms of its integrated roughness effect on the flow. The analysis of jets used in the disturbance region and reported [13] is a single WT wake model. The mentioned model was initially experimented on a large-scale data in Sweden [14]. The wind tunnel

modelling was also considered at the Netherlands Organization for Applied Scientific Research and at the Swedish National Aeronautical Research Institute [15–17]. The experimental data presented by Vermeulen and many reforms provided to the Lissaman model, which is now recognized as MILLY [18]. In the UK, different evaluations were run in a wind tunnel [19]. Regarding the single-wake in WTs, several research works were performed [20–25]. The point of view in the past research works focuses on the velocity deficiency but the recent studies mostly concentrate on the turbulence parameters. Many Danish researchers accomplished investigations of WFs. Initially, the wind and turbulence region near the two Nibe turbines were completely studied [20–23], and then the two large WFs Tændpibe [24] and Norrekaer Enge [20, 25] were investigated. In Netherland, a study of WF including 18.3 MW WTs has proposed many results of the flow structure in a WT cluster [26–28]. Since 1990, the results obtained have been derived at the Swedish Alsvik WF [29]. In the recent research works, Swedish Alsvik data have also been excavated. The mentioned studies can all be considered for the flat site locations but several research works have been performed for various topological areas such as USA WFs [30, 31] and Greek WFs [20]. Based on the information mentioned above, in addition to the referenced model [18] and other models that are influenced by [13], e.g. the Riso model [38], many models of the k-ε have been improved [32–35]. The WF layout design is a multi-disciplinary problem. Two major parameters, namely the anticipated power output and the wake influence, must be considered. The maximum power output can be achieved by minimizing the wake effect. Two important features of the wake modeling are the decline in the wind speed and increase in the level of disturbance of wind. Reference [36] solved WF optimizing the number and position of WTs using GA. They assumed a rectangular area consisting of 100 possible places for turbine locations using the Jensen’s wake model in order to analyze the wake effect under various wind speeds and directions. The improved results were achieved with a better GA in [37]. A research work in reference [45] followed the model that was indicated in [44] using MCm. The WF cost analysis based on the turbine rotor diameters and the turbine numbers was done by Chowdhury et al. [38] using the constrained Particle Swarm Optimization (PSO). Nested GA for WF with different hub height WTs was used by Chen et al. [39], resulting in a slightly more optimal design

compared to the case with identical hub WTs. Different cost models and hub height WTs were the main features of this work. Some researchers used the mathematical programming approaches. For example, Donovan [40] proposed the multiple mixed-integer linear models based on the vertex packing problem between a couple of WTs. In another paper, a new mathematical approach to optimize the layout of a 10 \* 10 grid of a possible turbine location in a WF was developed by Turner et al. [41], introducing the interaction matrices to model the wake effect between the turbines. The quadratic integer programming as well as the mixed-integer linear programming approaches were used. The computational fluid dynamics is not an efficient tool to calculate the velocity deficit due to the computational difficulties. The analytic and quasi-experimental wake models have been developed over the past decade. The velocity deficit modeled by these models generally correspond to the computational fluid dynamics simulations and wind tunnel measurements. In this regard, changing wind robustness quantification and layout optimization was considered by Feng et. al. [42]. Mike studied the WF multi-objective wake redirection for optimizing power production and loads [43]. The Gaussian-based wake model was another approach taken into account by Parade et al. [44]. Mirhassani et al. and Vassel-be-hagh et al. considered uncertainty and hub height in their optimization studies, respectively [45, 46].

**2. Methodology**

In this research work, the comparison of MCm simulation and GA in optimal WF layout design in the Manjil site is based on the Jensen model. A GA method is used in conjunction with MCm in order to obtain a favorable and optimized placement for Vestas V-47 WTs for maximum power output, while decreasing the number of WTs in WFs and the land acreage under cultivation of each WF. All the input data for the simulation is provided in Table 1.

**Table 1. Input data for simulation.**

Blade overall length in Vestas V-47	22.9 m
Rotor diameter in Vestas V-47	47 m
Hub height in Vestas V-47	40 m
Roughness in Manjil site	0.3
Manjil WF dimension	2000 × 2000 m <sup>2</sup>
Thrust coefficient	0.88

**2.1 Jensen model**

Jensen provided an efficient and suitable model for the turbine wake effect simulation, considering a negative jet. Based on the Jensen model, the rotational vortices are neglected near the blade.

The momentum deficit behind the turbines is idealized with a linear model. As a result, the turbines must be positioned far enough from each other both row wise and column wise. It is assumed that the wake radius is proportional to the distance downward the turbine. Thus from the momentum balance, equation (1) holds.

$$\pi r_0^2 u + \pi (r^2 + r_0^2) v_0 = \pi r^2 v_1 \tag{1}$$

Solving the above equation for  $v_1$  and considering the Betz theory for a flow speed behind the rotor, relation (2) holds.

$$v_1 = v_0 \left( 1 - \frac{2}{3} \left( \frac{r_0}{r_0 + \alpha x} \right) \right) \tag{2}$$

A conical shape approximates assuming linear expansion in the turbine wake, the profile of the flow passage. The conic radius is calculated by relation (3).

$$r = r_0 + \alpha x \tag{3}$$

In regular conditions,  $\alpha$  can be approximated by 0.1. Having used this assumption, one can compare the value of  $r$  to that of the experimental results considering suitable values for  $r_0$  and  $\alpha$ . Based on the above discussion, relation (4) is considered for the wake velocity.

$$u_{ij} = u_0 \left\{ 1 - \frac{2a}{1 + \alpha \left( \frac{x_{ij}}{R} \right)^2} \right\} \tag{4}$$

Where,  $\alpha$  is initially derived by Frandsen is shown in equation (5).

$$\alpha = \frac{0.5}{\ln \left( \frac{z}{z_0} \right)} \tag{5}$$

In the above equation,  $a$  is derived from the turbine thrust coefficient in relation (6), which should be less than 0.5.

$$C_T = 4a(1 - a) \tag{6}$$

The wake radius exactly behind the turbine is equal to the turbine diameter. The wake radius downstream can be defined in terms of the axial induction factor and turbine diameter in equation (7).

$$r_d = r_0 \sqrt{\frac{1 - a}{1 - 2a}} \tag{7}$$

The wake model is described in figure 1. The model proposed by Katic can estimate the wake effect for various conditions with an acceptable accuracy compared to the experiments.

The model is based on the flow kinetic energy that means the final kinetic energy loss is equal to the sum of the energy losses of all turbines, and is used in order to predict the flow velocity downstream of  $n$  turbines by equation (8).

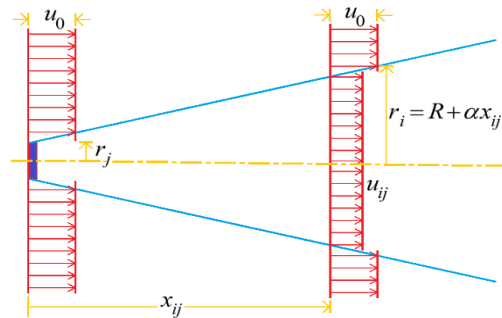


Figure 1. Jensen wake model.

$$\left( 1 - \frac{U}{U_0} \right)^2 = \sum_{i=1}^n \left( 1 - \frac{U_i}{U_0} \right)^2 \tag{8}$$

Figure 2 depicts the wake velocity in terms of the distance between two consecutive turbines.

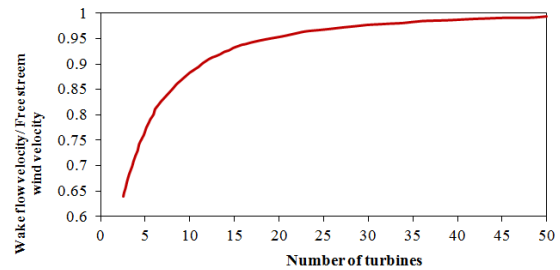


Figure 2. Wake velocity in terms of distance between two consecutive turbines.

As it can be seen, at a distance of about 50 times the turbine blade diameter, the wake flow velocity reaches the free stream wind velocity, which is not optimal.

### 2.2 Wake effects on turbine

Figure 3 displays the wake effect on the downstream turbines.

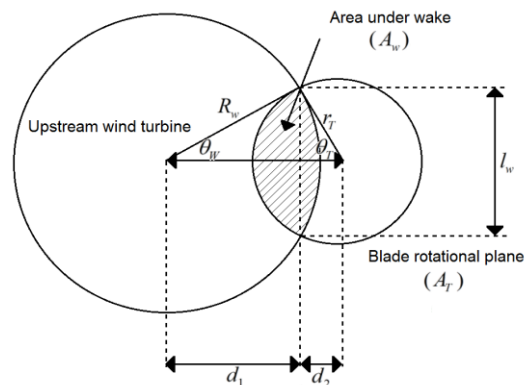


Figure 3. Wake effect on downstream turbines.

The shadowed region  $A_W$  is the partial area of the blade rotational plane in the downstream turbine,

$$A_W = R_W^2 \left( \theta_W - \frac{\sin(2\theta_W)}{2} \right) + r_T^2 \left( \theta_T - \frac{\sin(2\theta_T)}{2} \right); \forall X \in [R_W - r_T, R_W + r_T]$$

$$\begin{cases} A_W = 0; \forall X \geq R_W + r_T \\ A_W = A_T; \forall X \leq R_W - r_T \end{cases} \quad (9)$$

$$\theta_W = \cos^{-1} \left( \frac{R_W^2 + X^2 - r_T^2}{2XR_W} \right); \theta_T = \cos^{-1} \left( \frac{R_W^2 - X^2 - r_T^2}{2Xr_T} \right); X = d_1 + d_2$$

The turbine velocity affected by the partial wake is defined by equation (10).

$$\left( 1 - \frac{\bar{u}_i}{u_0} \right)^2 = \frac{A_W}{A_T} \left( 1 - \frac{u_{ij}}{u_0} \right)^2 \quad (10)$$

The following three conditions are considered:

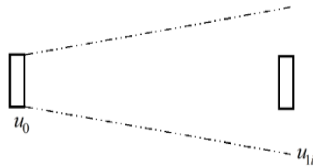
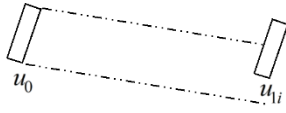
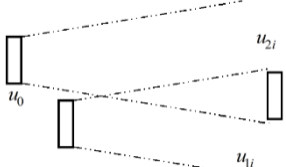
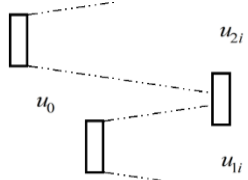
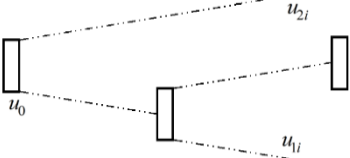
- The downstream turbine is completely affected by the upstream turbine wake, the condition under which the  $\frac{A_W}{A_T}$  parameter is equal to 1.

which is influenced by the upstream turbine wake.  $A_W$  can be calculated by equation (9).

- The downstream turbine is partially affected by the upstream turbine wake, the condition under which the  $\frac{A_W}{A_T}$  parameter is in the range of 0 to 1.
- The downstream turbine is outside the upstream turbine wake, the condition under which the  $\frac{A_W}{A_T}$  parameter is equal to 0.

The wake effect of an upstream turbine on the downstream units is depicted in figure 4.

Table 2. Different wake effect conditions and pertinent formulas.

Wake shape effect	Velocity decay formula
<p>Case 1: Complete wake</p> 	$u_i = u_{1i} = u_0 \left[ 1 - \left( \frac{2a}{[1 + \alpha(x_{ij}/r_i)]^2} \right) \right]$
<p>Case 2: PARTIAL WAKE</p> 	$\left( 1 - \frac{\bar{u}_i}{u_0} \right)^2 = \frac{A_W}{A_T} \left( 1 - \frac{u_{1i}}{u_0} \right)^2$
<p>Case 3: MULTIPLE TURBINE WAKE</p> 	$\left( 1 - \frac{\bar{u}_i}{u_0} \right)^2 = \left( 1 - \frac{u_{1i}}{u_0} \right)^2 + \left( 1 - \frac{u_{2i}}{u_0} \right)^2$
<p>Case 4: PARTIAL WAKE</p> 	$\left( 1 - \frac{\bar{u}_i}{u_0} \right)^2 = \frac{A_{W1}}{A_T} \left( 1 - \frac{u_{1i}}{u_0} \right)^2 + \frac{A_{W2}}{A_T} \left( 1 - \frac{u_{2i}}{u_0} \right)^2$
<p>Case 5: PARTIAL WAKE</p> 	$\left( 1 - \frac{\bar{u}_i}{u_0} \right)^2 = \frac{A_{W1}}{A_T} \left( 1 - \frac{u_{1i}}{u_0} \right)^2 + \left( 1 - \frac{A_{W1}}{A_T} \right) \left( 1 - \frac{u_{2i}}{u_0} \right)^2$

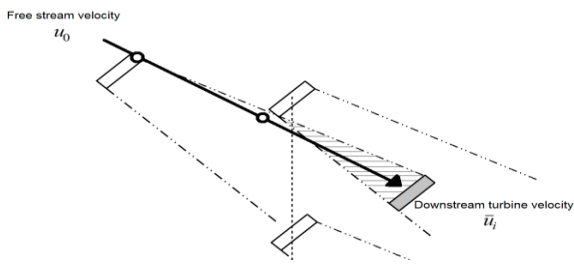


Figure 4. Effect of upstream turbine on downstream units.

### 2.3 Optimization

In this section, the arrangement of the horizontal axis wind turbines in the wind farm will be optimized. For this purpose, first, the genetic algorithm (GA) and Monte Carlo method (MCM) are explained, and then the objective function used in this research work will be defined.

#### 2.3.1. MCM

Two main methods can be utilized in order to analyze the large dimensional systems [55] such as the one in this case. First, the WF equipment location is defined for an analytical solution. In this way, important location conditions are considered to WF design criteria. Then MCM is used for a statistical study. MCM covers all the location conditions completely, with which the WT installation could be optimized. By random generator sampling, MCM calculates the results [56]. When the exact result is impossible to calculate, MCM can be used. When the physical phenomenon in WF is mathematically modeled and has uncertain parameters, this method is suitable for modeling this system, which has a high degree of freedom. In the MCM, the results are considered as parameters of a hypothetical community, an example of which can be constructed using a random numerical sequence from which a statistical estimation of the parameter can be obtained [57]. A flowchart of the MCM used in this work is shown in figure 5.

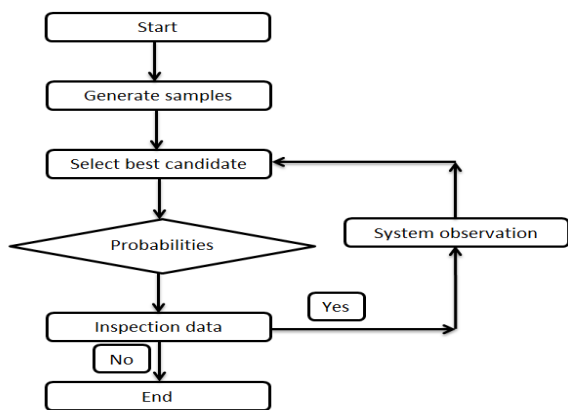


Figure 5. MCM flowchart.

#### 2.3.2. GA

GA in MATLAB is utilized in order to optimize the WTs location conditions in a WF. Primarily, GA randomly generates the binary chromosome strands. Each chromosome strand is an individual, which indicates the locations of WTs in WF. Selection, cross-over and mutation are the other three important stages in GA. After three main stages, the individuals with better results are passed on to the next generation, and the rest are eliminated at the same time. GA then replaces the deleted items with some new random ones in order to keep the same number of individuals in each generation. GA will continue uninterruptedly until it reaches the maximum number of production given. The recent studies [43, 44, 58, 59], have used WTs with a height of single hub, and they only had to consider the position of each WT in the desired WF so that a binary string in GA is enough to indicate the layout of a WF. A flowchart of the GA used in this work is shown in figure 6.

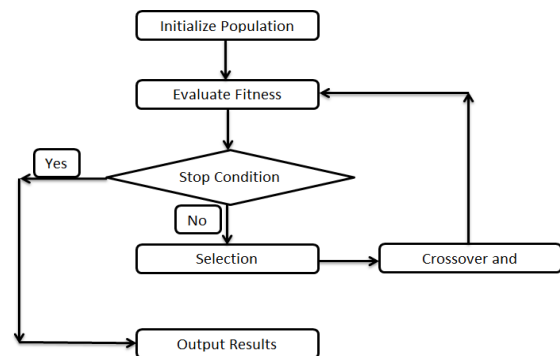


Figure 6. GA flowchart.

#### 2.3.3 Objective function

In order to define the objective function, a regular cost model is considered for the WF, which has been used in many research works. The cost function is dependent on the number of turbines installed in the WF. This non-dimensional function gives the cost involved in preparing and developing the WF. As it is suggested in the literature, the overall investment can be reduced by half.

The overall cost is defined by relation (11).

$$\text{Cost} = N_t \left( \frac{2}{3} + \frac{1}{3} e^{-0.00174N_t^2} \right) \quad (11)$$

Figure 7 depicts the cost function in terms of the number of turbines, and figure 8 shows the derivative of the cost function in terms of the number of turbines.

It is noticeable from the above figure that the function minimum is placed between the number

of turbines 29 and 30. After reaching the minimum, the derivative follows a rising trend, which means for a turbine number greater than 30; the cost of N+1 turbine is more than N turbine. Thus a further increase in the turbine number would not be efficient. Since only the WF cost was considered, this result only holds with respect to the cost specifications.

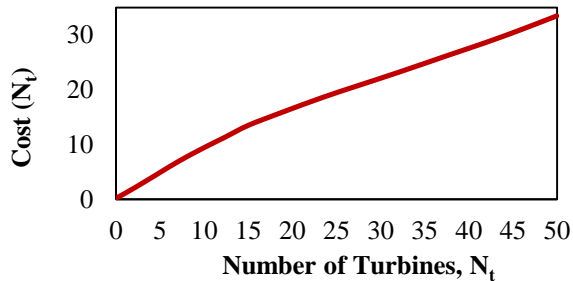


Figure 7. Cost function vs. number of turbines.

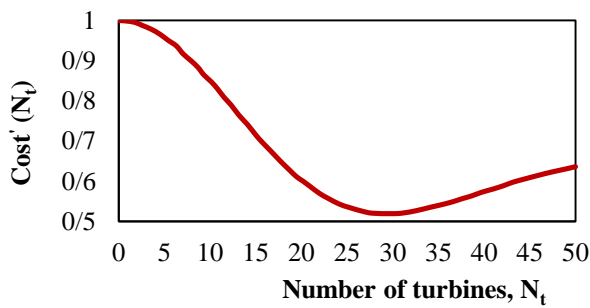


Figure 8. Cost function 1<sup>st</sup> derivative vs. number of turbines.

Relation (12) holds for the WF available power production.

$$\text{Available power} = \frac{1}{2} \rho A u_t^3 \quad (12)$$

The blade rotational plane area is equal to  $\pi R^2_b$ . The above relation is for an ideal condition, and an efficiency factor must be multiplied where it is shown in equation (13).

$$\text{Produced power} = \eta \frac{1}{2} \rho A u_t^3 \quad (13)$$

$\eta$  is typically equal to 0.4, and the standard air density is 1.2. Therefore, Equation (13) becomes equation (14).

$$\text{Produced power} = 0.24 A u_t^3 \quad (14)$$

During the calculation process, the number of turbines by which the downstream turbine is influenced should be checked. The partial condition of the wake should also be considered. The higher the number of upstream turbines, the greater the wake loss will be.

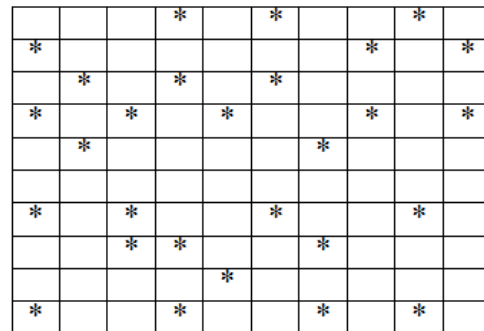
The final objective function is formulated as equation (15).

$$\text{Objective function} = \frac{N_t \left( \frac{2}{3} + \frac{1}{3} e^{-0.00174 N_t^2} \right)}{0.24 A \sum_{i=1}^{N_t} u_i^3} \quad (15)$$

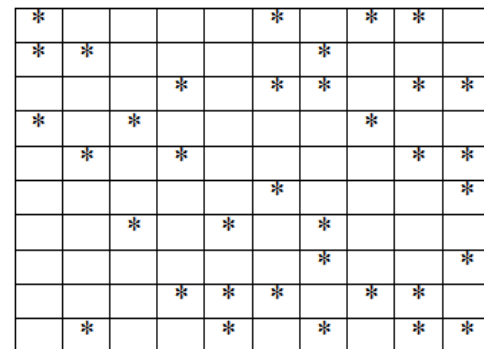
The numerator is the investment cost, and the denominator is the production power of the WF, which is dependent on the turbine placement, and must be calculated for each turbine based on their positions.

### 3. Result and discussion

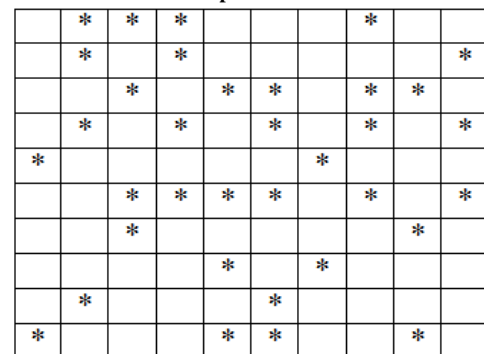
In this work, the Vestas V47 horizontal WT was arranged in a WF on a square ground sub-divided into 100 (10m\*10m) turbine locations so the turbines will be 5D apart. In the GA, 600 individuals initially gather in more than 20 sub-populations, and evolve into more than 9,000 generations. Since the wind velocity in our studied wind field varies between 10 and 15 m/s, the results for different wind velocities (10, 12, and 15 m/s) are provided (Figure 9).



Wind speed of 10 m/s



Wind speed of 12 m/s



Wind speed of 15 m/s

Figure 9. GA results for different wind speeds.

In the case of MCm 600 individuals spreading over 20 sub-populations evolve for 5000 generations. Since the wind velocity in our studied wind field varies between 10 and 15 m/s, the results for different wind velocities (10, 12, and 15 m/s) are provided (Figure 10).

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10000 samplings, wind speed 15 m/s

Figure 10. MCm results for different wind speeds.

A comparison between the presented results and the previous works is provided in table 3 and figure 11.

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Figure 11. Display of the result of four studies (a) Our results (2021). (b) Grady et al.'s (2005) [44]. (c) Marmidis et al.'s (2008) [45]. (d) Mossetti et al. (1994) [43].



**Table 3. Results for wind farm layout.**

	Number of turbines	Total power (KW/year)	Fitness value
Present paper-GA	28	13127	0.001604
Present paper-MC	36	18387	0.001374
Marmidis	32	16395	0.0014107
Grady	30	14310	0.0015436
Mosetti	26	12352	0.0016197

Table 4 shows the proposed optimization results of the presented work and the other three studies as the number of turbines, the power output, and the fitness value. The first two rows show the results obtained in the presented work. Although the GA result is not better than the previous work, the MCm result is better than all cases, as our total power output is 18.387 MW/year compared to the 12.352 MW/year in Ref. [43], 16.395 MW/year in Ref. [45], and 14.310 MW/year in Ref. [44]. The critical factor compared to the other studies is the fitness value that will specify whether the result of this optimization is optimal or not. The fitness value is defined as the ratio between the cost and the power output. The fitness value is the critical factor as equation (16), which specifies whether this optimization is optimal or not, in comparison with the other studies.

$$\text{Fitness value} = \frac{\text{Cost}}{P_{\text{total}}} \quad (16)$$

In which the cost is defined by equation (11). Mosetti et al. reached a 0.0016197 fitness value, Grady et al. 0.0015436, and Marmidis 0.0014107. The fitness value of the present work, which is as smallest as 0.001374, demonstrates that the results of the present work are more favorable and optimal. Conclusively, we not only reached the greatest number of turbines and the greatest total power, we also obtained greater prices of the fitness value. The presented placements of all the discussed studies are shown in figure 11.

#### 4. Conclusions

In this work, the limitations of WT installation and conventional optimization methods for achieving an optimal WF were investigated. The background of this work was thoroughly investigated. In order to validate the developed codes, three studies in this field were compared with the present results. Finally, optimization was performed for the defined function using both GA and the MCm algorithm.

The results obtained show that MCm provides a better optimization result since we reached a greater number of turbines, a greater total power, and even a lower of fitness value. Under the same conditions, the Monte Carlo algorithm gave 29%

and 40% better results in terms of the number of turbines and the output power, respectively. In terms of optimization, in the Monte Carlo algorithm, its fitness value was 16% less than the genetic algorithm, which indicates its better optimization.

This work can be considered as a single type of turbines but it can be applicable if different types of turbines are available. The first population of the present work is completely random, and may achieve a better result in less time if the first generation is developed appropriately.

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