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Experimental Analysis and Parametric Optimization of Linseed Oil Methyl Ester Blend-Fueled Variable Compression Ratio Diesel Engine

Ganesh S. Warkhade[1] Katam Ganesh Babu[2*] and A. Veeresh Babu[3]

1. Department of Mechanical Engineering, Dr. Babasaheb Ambedkar Technological, Lonere (MS), India.
2. Department of Mechanical Engineering, National Institute of Technology Andhra Pradesh, TadepalliGudem (AP), India.
3 Department of Mechanical Engineering, National Institute of Technology, Warangal-506004, Telangana, India

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**Corresponding author: katamganeshbabu@gmail.com (Ganesh Babu Katam)*

Abstract
This paper analyses the VCR (variable compression ratio) engine's performance, combustion, and emission output responses. The experimental results are modelled using the Grey Taguchi method (GTM) for the input parameters of compression ratio, load, and fuel blends. The objective is to find the optimal combination of the input parameters in the minimum number of experiments for minimum emission, better performance, and combustion parameters. The Taguchi's L9 orthogonal array with GTM is used to get the optimum combination of the input parameters. The Taguchi is used to analyze the S/N ratio of the experimental data and the gray-based method for optimization of the multi-objective to single-objective optimization by assigning the suitable weighting factor to each response. The S/N ratio analysis of grey relational grade (GRG) shows the fuel B10, CR 16, and load at 100% of the optimal input factor level. This optimal level is further confirmed by the TOPSIS method. The analysis of variance (ANOVA) for input to GRG shows the highest influencing factor is the load with a 52.82% contribution, followed by CR at 28.38%, and fuel at 10.52%. The confirmatory results show an improvement of 56.1%. The novelty of this experimentation is to study the feasibility of existing engine for alternative fuel with a slight modification. At the above optimal conditions, this biodiesel can be used efficiently in an unmodified compression ignition engine.

Keywords: *Combustion, performance, emission, biodiesel, priority matrix, GTM, TOPSIS.*

1. Introduction

In every field of industrialization, the diesel engines are used due their high reliability, thermal efficiency, excellent power, and fuel efficiency (40-50%) over the spark ignition engines. Consequently, an industrial and economic development is attributed to air pollution, which causes 6.5 million deaths each year [1]. This alarming level of air pollution and depleting fossil fuel mainly is due to the excessive use of petroleum products in automotive industries. Moreover, energy demand for the world up to 2030 would be 50% more than today, out of which, the US, China, Japan, and India are consuming more energy as compared to the other countries [2]. In 1990, India was importing the oil products around 43%, and that increased to 71% in 2012, as a result, a lot of burden on the Indian economy. Out of total energy consumption in India, 51% share is of the transport sector, and this consumption rate estimated will be raised by 6 to 8 percent shortly [3]. The ever increasing demand for fossil fuel causes declining its sources, and also higher pollution level leads to strict emission regulations for the transport vehicles. This feature has forced the decision-makers and researchers to explore a fuel that produces minimum pollution as well as renewable and sustainable like vegetable oil based biodiesel, alcohols, etc. to replace the conventional fuel shortly. The vegetable oil based biodiesel could be the first generation, i.e. edible oil such as coconut, palm, rapeseed, sunflower, and second generation (non-edible oil) such as Pongamia pinnata, jatropha curcus, castor, sea mango, neem, and mahua, and third generation biodiesel based on microalgae[4]. The second or third generation biodiesels does not affect the food security, and hence, mostly selected for biodiesel production. Therefore, to encourage the agriculture sector, and to ease the fuel shortage, and burden on the economy, India has proposed for blending the 20% biodiesel and alcohol in transportation fuel by 2017 under the national biofuels policy [5]. Biodiesel's many salient features viz. biodegradable nature, not any traces of sulfur and aromatic compounds can reduce the toxic emissions. These excellent characteristics of biodiesel could prove the good option to use the biodiesel-fueled engine in urban and no pollution zone area namely in parks. In the literature, it mentioned that by mixing a small amount of biodiesel can enhance fuel viscosity, extend engine life, and increase fuel efficiency.

1.1 Engine optimization literature

The all existing transport and industrial diesel engines are designed and optimized, considering fuel as petroleum diesel. Therefore, the operating parameters have to be optimized to use the biodiesel as fuel in the unmodified engine to reduce the design and modification cost. Many researchers have studied the optimization of compression ignition engine using different biodiesel/diesel fuel blend. The various techniques used for optimization of engine parameters namely response surface methodology (RSM) [6,7–9,10], Taguchi approach [11,12,13], Taguchi-grey relational analysis method (GTM) [14,15,16], GTM-TOPSIS(technique for order preference by similarity to ideal solution) method [17], genetic algorithm and artificial neural network [18]. These techniques are helpful for reliability and accuracy to get the result in a minimum number of experiments and cost. Parameter optimization of Karanja biodiesel/diesel-fueled engine was carried out using non-linear regression and reported the optimized factor of fuel blend B13 and IT at 24 ^{0}bTDC [19]. Thermodynamic and Taguchi model used to analyze the Jatropha biodiesel fed engine. The maximum engine performance for biodiesel found at optimized engine design and performance parameters [20].

The algorithm based on particle swarm optimization (PSO) was used to investigate the diesel engine parameters for improving the fuel efficiency and decrease the engine out emissions [21]. The optimization of performance and emission parameters was carried out in a single cylinder diesel engine of power 5.2 kW fueled with Karanja biodiesel. The input parameters were compression ratio, fuel fraction, injection timing (IT), injection pressure (IP), and load with output parameters fuel consumption, emission, and brake power. The input parameters were taken at 4-level and Taguchi-GM approach used with orthogonal array L_{16} for getting optimum settings. The authors reported the compression ratio 17.7, brake power 3.64 kW, biodiesel blend B20, IT 27 ^{0}bTDC, and IP 230 bar optimum operating parameters of the engine [22]. Fish oil biodiesel blends and a linseed oil biodiesel blends a load on the engine was varied at six and five levels, respectively. The optimization of various performance, combustion, and emission parameters has been done using fish oil biodiesel. The Taguchi-fuzzy approach was used to optimize the parameters, and ANOVA was used to get the effect of each working parameter on output parameters [23]. An experimental study was carried out in a dual fuel mode of CNG-diesel and optimizations of IP, load, and energy shared by CNG (CES) have been done to reduce the brake specific fuel consumption, emission of net hydrocarbon (NHC) and particulate matter (PM). Using grey-Taguchi approach, they reported the optimal input parameters as load 4 kg, IP 540 bar, and CES of 15% [24]. Zhan-Yi Wu et al. [25] investigated the combustion and emission features at optimal operating condition using Taguchi approach. They noted the biodiesel fuel blend B10, liquid petroleum gas (LPG) 40%, EGR ratio 20%, and load 60% as the optimal operating parameter for reducing the emission of smoke by 52% and NOx 31%. The literature shows that most of the optimization of the engine parameter, evaluated with biodiesel of Karanja, Jatropha, Mangifera indica, Pongamia, mahua, fish oil, honge, and sesame and that too mostly on performance and emission parameters. Furthermore, it shows that each biodiesel has a different optimized parameter [26] for maximum performance and minimum emission characteristics of compression ignition engine. This different optimum setting for each biodiesel depends on their physical and chemical properties. In this connection, still, much more non-edible oil's biodiesel has to be studied for its optimization for using in an unmodified engine. The available information on optimization related to combustion responses and linseed methyl ester is less to the best of author's knowledge.

The design of experiment's (DOE) Taguchi method used for the analysis purpose. This method proposed by the Dr. Genichi Taguchi [27] for optimization of the parameter, which provides the information about the best control parameters in the least number of experiments. The accuracy and reliability of the Taguchi method solely depend upon the way the factors and their values have been chosen. In the Taguchi design, the robustness of any control elements is measured by the way it affected by the uncontrolled factors (noise level). The purpose of

Taguchi design is to identify the best control factor, which has a less variability due to the uncontrolled factors (noise level) such as ambient temperature, and engine vibration. The variability in control factors is measured by the Taguchi's signal to noise ratio (S/N). The S/N ratio is the log function of output measured parameter, and these are to be calculated for each output parameter. The higher the S/N ratio means better the control factor and less variability due to the noise levels [28,29]. The S/N ratio calculated by the three design conditions namely larger the better, smaller the better, and nominal the better. The arrangement of control factors and their levels in a minimum number of experiments called orthogonal array to get the effect of control factors on given responses. However, the Taguchi method is used for single objective optimization. For more than one responses/multi objective (output parameters), the grey relational analysis method proposed by Deng [30] is used. In the GM method, all the responses are combined and converted into a single response optimization problem. The Taguchi and GM methods are combined for optimization of multi-objective responses.

The grey relational analysis method (GM) concept uses two conditions of information. The condition at which not at all any information (black) is available for the system ultimately, there is no solution. Another side is with full of information (white), which could have a unique solution for the system of information. However, these kinds of extremities never exist in real world but somewhere in between. Therefore, GM uses to solve the problems that have less or partially available information. That converts the multi-objective problem into single objective and Taguchi used for optimization. Many authors have used this combined technique of optimization for solving the problems [31,32–35, 36].

The TOPSIS method has been used in many applications for optimization purpose. However, the authors [37] noted its limitations as it does not accounts relative significance of distances it measures from two reference points for optimal solution. Furthermore, it has been reported that the TOPSIS and grey relational methods have the similarity and the limitation of TOPSIS can be overcome by integrating with GTM [38,39]. The optimal combination of input parameters has been obtained for performance improvement and reducing the emissions.

The objective of the present work is to optimize the combination of fuel blend, CR, and a load of linseed methyl ester/petro-diesel fed single cylinder diesel engine to improve the performance, combustion and to reduce the emission characteristics. This has been done by performing the experiments and with help of optimization tools. An orthogonal array L_9 used to arrange the input factors and their levels in nine numbers of experiments. GTM has been used to optimize the responses and to identify the best combination of input parameters. The Minitab 17® software has been used for analysis of the experimental results. Further, the method of TOPSIS confirms the optimized results of GTM. The novelty of this experimentation was to study the feasibility of existing engine for alternative fuel with slight modification and confirmation of operating/fuel parameters.

2. Materials and experimental methodology

2.1. Fuel preparation:

Linseed oil was selected for the present work; India is the third major producer of linseed oil after Australia and Canada. Linseed is mainly cultivated in India for oil and fibers in the month of October–November. Linseed Seed contains around 33% to 47% of the oil. Its seed cake used for feeding a cattle and a small part of its oil used for edible and most of the linseed oil (80%) used for industrial purpose. This oil highly unsaturated and has more percentage of linolenic acid, and therefore, it can be employed for making oil cloth, paints, printed ink, and varnish, etc. There are different methods for biodiesel preparation. Those are pyrolysis, macro-emulsion, dilution, and transesterification. Transesterification is the most widely used process because it is simple, efficient, and economical. Here, two stage transesterifications are used to convert the raw oil into biodiesel. The transesterification process uses the catalyst for carrying out the reaction and could be base, acid, enzyme. In this process, initially, esterification of raw oil is performed followed by the transesterification of esterified oil. Thus the two stages are esterification followed by transesterification. Esterified linseed oil was obtained from esterification reaction, i.e. first step of two-stage transesterification of biodiesel preparation method. In the esterification reaction, the reaction takes place between the carboxylic acid group (free fatty acid) present in the fresh linseed oil and with the alcohol in the presence of an acid (H_2SO_4) catalyst. In this reaction, –OH from the carboxylic acid combine with –H from alcohol and produce an ester of linseed oil and H_2O as a by-product, as given in equation 1 [40, 41].

$$\underset{FFA}{R-COOH}+\underset{Alcohol}{R_4OH}\xleftrightarrow[Catalyst]{H_2SO_4}\underset{Water}{H_2O}+\underset{Ester}{R-CO-OH_2C-R_4} \quad (1)$$

The esterification reaction does not remove the glycerol and adds additional carbon chain. This esterified linseed oil (ELO) is treated with methanol and catalyst (NaOH) with different combinations for getting the optimum yield as shown in table 1. Then the quantity of methanol (20% vol.) and catalyst (0.5% weight) has been used for mass production of biodiesel. The prepared linseed methyl ester biodiesel (LB) and its blends with petro-diesel have been given in table 2. The different volumes of biodiesel have been mixed with petro-diesel to prepare the biodiesel fuel blend; LB10 indicates 10% of biodiesel on the volume basis and remaining petro-diesel volume.

Table 1. Biodiesel conversion efficiency.

Trial	ELO (mL)	Methanol (vol. %)	Catalyst (wt. %)	Yield (vol. %)
1	200	20	0.35	80
2	200	20	0.36	80
3	200	20	0.39	75
4	200	20	0.4	66
5	200	20	0.5	85

Table 2. Fuel properties.

Properties	Petrodiesel	LB10	LB20	LB30
Density at 40^{0}C (kg/m^3)	829	834	842	853
Kinematic viscosity at 40^{0}C (cSt)	2.68	2.82	3.12	3.3
Flash point (^{0}C)	50	96	102	120
Fire point (^{0}C)	--	102	110	125
Lower Calorific value (MJ/kg)	43.5	42.65	42.11	40.83

2.2 Experimental setup and methodology

The experimental setup's schematic diagram is shown in figure 1. The experiment has been conducted with single cylinder four strokes; variable compression ratio diesel engine. The engine has 3.5 kW power rating at 1500 rpm, bore diameter of 87.5 mm, and stroke length of 110 mm. The capacity of the engine is 0.661 liters with connecting rod length of 234 mm.

F_1 = Fuel flow, F_2 = Air flow, F_3 = Engine cooling water flow, F_4 = Calorimeter water flow, N = Speed of engine in rpm, W = Eddy current dynamometer, PT = Pressure transducer, DAS = Data acquisition system, EGA = Exhaust gas analyzer T_1 = Engine inlet cooling water temp. T_2 = Engine outlet cooling water temp, T_3 = Engine exhaust gas temp.

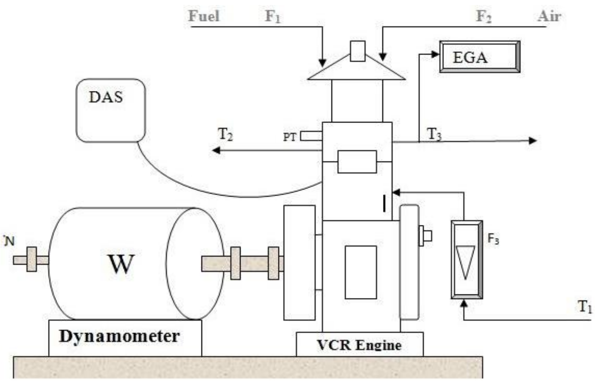

Fig 1. Schematic diagram of experimental setup.

The experimental setup has been provided with suitable instruments for combustion pressure, fuel line pressure, and crank angle measurements. The measured data is interfaced with a computer for generating the pressure crank angle diagrams. The panel box of the engine set up consisting of digital temperature indicators, digital voltage indicator, and digital load indicator with load control knob of eddy current dynamometer, air-box with orifice meter for measurement of air flow and graduated glass burette for fuel flow indicator. All these experimental data signals are transmitted to a Labview-based data acquisition software "ICEnginesoft" for online performance/ combustion evaluation. For loading the engine, the strain gauge type load sensor of range 0-50 kg used. The temperature sensors RTD, PT100, and K type thermocouples are used to measure exhaust gas and engine cooling water temperature at various locations with accuracies of ±0.1 ^{0}C. Engine water flow rate was adjusted at 300 mph by using the suitable Rotameter to maintain the engine cooling water temperature 70 ^{0}C at the outlet. The combustion pressure and fuel line pressure are measured by using the two piezoelectric sensors of range 0–5000 PSI; crank angle sensor is used for measuring the crank angle with a resolution of 1 degree of crank angle. The emission parameters measured at variable CR and load using INDUS five gas analyzer (PEA 205N, Make: INDUS). The specification of gas analyzer has tabulated in table 3.

Table 3. INDUS (PEA 205N) Five Gas Analyzer specification.

Measured	Range	Resolution	Accuracy
CO	0 to 15% Vol	0.01% Vol	±0.02% Vol; ±3% O.M
CO_2	0 to 20% Vol	0.01% Vol	±0.3% Vol; ±3% O.M
HC	0 to 30000 ppm	≤ 2.000: 1 ppm vol.	< 2000 ppm vol.: ±4 ppm vol. ±3 O.M.
O_2	0 to 25%	0.01% vol.	± 0.02% vol.
NO_x	0 to 5000 ppm	1 ppm vol.	± 5 ppm vol.

The rated power rating of the engine is 3.5 kW at 1500 rpm with water cooled eddy current dynamometer for loading the engine. The engine has the suitable sensing device for temperature, pressure, flow rate, and crank angle measurements with data acquisition system. The engine has compression ratio (CR) ranges from 12:1 to 18:1. In the present work, the CR selected was 14:1, 16:1, and 18:1. For each CR, the load has been varied from 60% to 100% (7.4 to 12.28 kg), in the step of 20%. The combustion, performance, and emission characteristics were observed for each load and CR. Injection pressure and timing were kept constant at 210 bar and 23^0 bTDC for all observations. Rigorous warming experimental work was performed. In each test, the engine was run for 5 minutes for properly up the engine and stabilizing the set of all working parameters. For reliability and accuracy, a set of results were taken six times for ten cycles each, and the best result is taken for analysis purpose.

2.3 Error /uncertainty analysis

Errors and uncertainties in the experiments can arise from instrument selection, condition, calibration, environment, observation, reading and test planning. Uncertainty analysis is needed to prove the accuracy of the experiments [42]. The percentage uncertainties of various parameters like brake power and brake thermal efficiency were calculated using the percentage uncertainties of various instruments given in table 4. An uncertainly analysis was performed using.

Percentage Error = Square root of sum of squares of the uncertainty in measuring instruments.

Percentage of uncertainty occurring in the experiments = square root of ((uncertainty of pressure transducer)^2 + (uncertainty of angle encoder)^2 + (uncertainty of NOx)^2 + (uncertainty of HC)^2 + (uncertainty of CO)^2 + (uncertainty of CO_2)^2 + (uncertainty of O_2)^2 + (uncertainty of Smoke opacity)^2 + (uncertainty K-2 thermocouple)^2 + (uncertainty of stop watch)2 + (uncertainty of manometer)^2 + (uncertainty of burette)^2)

%Error in experimental measurement = square root of $((0.1)^2 + (0.2)^2 + (0.2)^2 + (0.2)^2 + (0.6)^2 + (0.5)^2 + (0.01)^2 + (0.01)^2 + (0.15)^2 + (0.2)^2 + (1)^2 + (1)^2)$ = **1.674%**

3. Taguchi and GM technique for optimization

As mentioned, these two methods were combined to solve the multi-objective related problem. This combined method's steps are shown in figure 2. The left part of the figure indicates the Taguchi method and the right part is the GM method.

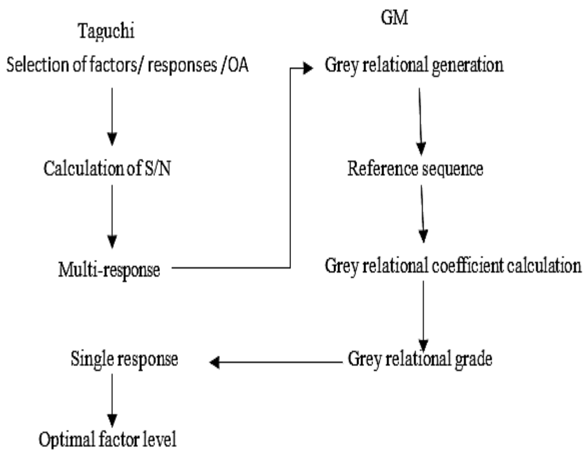

Figure 2. Steps in GTM method.

3.1. Selection of factors and their levels

The selection of factors and levels for optimization entirely depends on the designer's level of understanding the experimental setup and its effects on the output responses. In this study, the three input factors viz. fuel blend, CR, and load and their three levels have been selected, as shown in table 4.

Table 4. Factors and their levels.

Factors	Level 1	Level 2	Level 3
A: Fuel blend	B10	B20	B30
B: CR	14	16	18
C: Load (%)	60	80	100

These selected factors and levels are provided in a Taguchi's orthogonal array (OA) in such a way that optimization should be in a minimum number of experiments/trials [43,44,45]. These figures of the testing are calculated as per the equation 2.

$$\textbf{Minimum number of trials} = [(F - 1)\times L] + 1 = [(3 - 1)\times 3] + 1 = 9 = L_9 \quad (2)$$

where F and L are the number of factors and levels, respectively, selected for the study.

Based on the factors and their levels, these are arranged in a minimum number of trials (OA L_9). These combinations along with their experimental results of responses are given the table 5.

Table 5. Arrangement of factors and levels in orthogonal array (L_9) with experimental results.

Expt No.	OA L_9			Output parameters/ Responses									
	Fuel	CR	Load (%)	BTE (%)	BSFC (kg/kWh)	EGT (^{0}C)	CP (bar)	NHR (J/deg)	RPR (bar/deg)	CO %	HC ppm	NOx ppm	Smoke (HSU)
1	B10	14	60	20.93	0.4	302.418	30.74	29.19	0.84	0.09	37	511	17.64
2	B20	14	80	24.06	0.34	366.487	29.5	30.46	0.79	0.089	37	612	22.95
3	B30	14	100	26.14	0.34	422.33	30.41	36.77	1.43	0.174	52	710	41.88
4	B10	16	80	24.8	0.34	338.459	38.73	38.92	2.05	0.057	22	627	14.89
5	B20	16	100	25.67	0.33	402.296	39.43	38.39	1.89	0.114	31	701	31.02
6	B30	16	60	22.71	0.37	280.6	37.76	32.8	1.53	0.073	20	491	13.37
7	B10	18	100	28.85	0.3	350.928	46.99	33.89	2.13	0.036	13	893	20.63
8	B20	18	60	23.8	0.36	266.952	45.38	31.07	1.71	0.038	12	573	17.15
9	B30	18	80	28.31	0.31	300.26	45.9	28.15	1.13	0.042	15	755	19.91

3.2. Signal to noise (S/N) ratio calculation

The analyses of results are carried out by calculating the S/N ratio. In this study, for calculation of S/N ratios following two design conditions are used.

For larger the better characteristics

$$\boldsymbol{\eta}_{\text{ij}}= -10\times\log\left(\frac{1}{r}\sum_{k=1}^{r}\frac{1}{m_{\text{ijk}}^{2}}\right) \tag{3}$$

For smaller the better characteristics

$$\boldsymbol{\eta}_{\text{ij}}= -10\times\log\left(\frac{1}{r}\sum_{k=1}^{r}m_{ijk}^{2}\right) \tag{4}$$

Where η_{ij} is the S/N ratio of experiment number i for response j, and m_{ijk} is the simulation result for trial i for response j, in kth number of replication and r is the number of replication required. The BTE, CP, NHR, and RPR taken as 'larger the better' responses and BSFC, EGT, CO, HC, NOx, smoke are 'smaller the better' responses. The S/N ratio calculated by using the equations 3 and 4 and represented in table 6.

Table 6. Signal to noise (S/N) ratio of experimental results.

Expt. No.	S/N ratio									
	BTE	BSFC	EGT	CP	NHR	RPR	CO	HC	NOx	Smoke
1	26.42	7.96	-49.61	29.75	29.30	-1.51	20.92	-31.36	-54.17	-24.93
2	27.63	9.37	-51.28	29.40	29.67	-2.05	21.01	-31.36	-55.74	-27.22
3	28.35	9.37	-52.51	29.66	31.31	3.11	15.19	-34.32	-57.03	-32.44
4	27.89	9.37	-50.59	31.76	31.80	6.24	24.88	-26.85	-55.95	-23.46
5	28.19	9.63	-52.09	31.92	31.68	5.53	18.86	-29.83	-56.91	-29.83
6	27.12	8.64	-48.96	31.54	30.32	3.69	22.73	-26.02	-53.82	-22.52
7	29.20	10.46	-50.90	33.44	30.60	6.57	28.87	-22.28	-59.02	-26.29
8	27.53	8.87	-48.53	33.14	29.85	4.66	28.40	-21.58	-55.16	-24.69
9	29.04	10.17	-49.55	33.24	28.99	1.06	27.54	-23.52	-57.56	-25.98

3.3. Grey relational generation

The GM optimization was used to solve the multi-interdependent responses problem [46], the steps are shown in lower part of the flowchart of figure 2. In this part of optimization, the first step is to linear normalization of calculated S/N ratio between 0 and 1, known as grey relation generation.

The grey relation generation s_{ij} for trial i and response j has been calculated using equations 5 and 6. Equation 5 is used for larger the better responses and 6 for smaller the better responses for calculating the grey relational generation.

$$s_{ij} = \frac{\eta_{ij} - \min_j \eta_{ij}}{\max_j \eta_{ij} - \min_j \eta_{ij}} \tag{5}$$

$$s_{ij} = \frac{\max_j \eta_{ij} - \eta_{ij}}{\max_j \eta_{ij} - \min_j \eta_{ij}} \tag{6}$$

The grey relational generations for normalized S/N ratio are tabulated in table 7.

After calculating the grey relational generation, all the performance values are scaled up between 0 and 1. If the performance value s_{ij} for experiment number i of response j is 1 or nearer to 1, then this performance value of i is best for response j. However, these kinds of situations never exist; hence, a reference sequence X_0 (best/ideal value) = (X_{01}, X_{02}, ….) = (1, 1, ….) is introduced for comparability.

3.3. Calculation of grey relational coefficient

The grey relational generation compared with reference sequence and determined how close s_{ij} to X_0. This closeness is represented by the grey relational coefficient ω_{ij} and calculated as given in equation 7.

$$\omega_{ij} = \frac{\Delta_{\min} + \zeta . \Delta_{\max}}{\Delta_{ij} + \zeta . \Delta_{\max}}$$

$$\Delta_{\min} = Min\left(\Delta_{ij}, i = 1, 2, \ldots m; j = 1, 2, \ldots n\right) \quad (7)$$

$$\Delta_{\max} = Max\left(\Delta_{ij}, i = 1, 2, \ldots m; j = 1, 2, \ldots n\right)$$

where $\Delta_{ij} = |x_{0j} - s_{ij}|$ and ζ is the distinguishing coefficient used for compressing or expanding the range of ω_{ij} responses. m and n are the number of trials/experiments and responses.

The value of ζ lies between 0 and 1, and most of the researchers have taken the value of it as 0.5. However, any value of it does not affect the ranking of an optimum experimental alternative. The calculated values of Δ_{ij}, Δ_{min} and Δ_{max} are tabulated in table 8 and grey relational coefficient in table 9.

Table 7. Grey relational generations.

Exp. No.	BTE	BSFC	EGT	CP	NHR	RPR	CO	HC	NOx	Smoke
X_0	1	1	1	1	1	1	1	1	1	1
1	0.00	1.00	0.73	0.09	0.11	0.08	0.5816	0.2321	0.9333	0.7573
2	0.43	0.44	0.31	0.00	0.24	0.18	0.5745	0.2321	0.6317	0.5268
3	0.69	0.44	0.00	0.07	0.82	0.37	1.0000	0.0000	0.3834	0.0000
4	0.53	0.44	0.48	0.58	1.00	0.94	0.2917	0.5866	0.5912	0.9057
5	0.64	0.33	0.11	0.62	0.96	0.81	0.7316	0.3528	0.4047	0.2629
6	0.25	0.73	0.89	0.53	0.47	0.48	0.4487	0.6516	1.0000	1.0000
7	1.00	0.00	0.40	1.00	0.57	1.00	0.0000	0.9454	0.0000	0.6201
8	0.40	0.63	1.00	0.93	0.30	0.65	0.0343	1.0000	0.7418	0.7819
9	0.94	0.11	0.74	0.95	0.00	0.00	0.0978	0.8478	0.2807	0.6512

Table 8. Calculation of Δ_{ij}.

ExpNo.	BTE	BSFC	EGT	CP	NHR	RPR	CO	HC	NOx	Smoke
1	1.00	0.00	0.27	0.91	0.89	0.92	0.42	0.77	0.07	0.24
2	0.57	0.56	0.69	1.00	0.76	0.82	0.43	0.77	0.37	0.47
3	0.31	0.56	1.00	0.93	0.18	0.63	0.00	1.00	0.62	1.00
4	0.47	0.56	0.52	0.42	0.00	0.06	0.71	0.41	0.41	0.09
5	0.36	0.67	0.89	0.38	0.04	0.19	0.27	0.65	0.60	0.74
6	0.75	0.27	0.11	0.47	0.53	0.52	0.55	0.35	0.00	0.00
7	0.00	1.00	0.60	0.00	0.43	0.00	1.00	0.05	1.00	0.38
8	0.60	0.37	0.00	0.07	0.70	0.35	0.97	0.00	0.26	0.22
9	0.06	0.89	0.26	0.05	1.00	1.00	0.90	0.15	0.72	0.35
Δ_{min}	0	0	0	0	0	0	0	0	0	0
Δ_{max}	1	1	1	1	1	1	1	1	1	1

Table 9. Calculation of grey relational coefficient.

ExpNo	BTE	BSFC	EGT	CP	NHR	RPR	CO	HC	NOx	Smoke
1	0.33	1.00	0.65	0.35	0.36	0.35	0.54	0.39	0.88	0.67
2	0.47	0.47	0.42	0.33	0.40	0.38	0.54	0.39	0.58	0.51
3	0.62	0.47	0.33	0.35	0.74	0.44	1.00	0.33	0.45	0.33
4	0.51	0.47	0.49	0.55	1.00	0.89	0.41	0.55	0.55	0.84
5	0.58	0.43	0.36	0.57	0.92	0.73	0.65	0.44	0.46	0.40
6	0.40	0.65	0.82	0.52	0.49	0.49	0.48	0.59	1.00	1.00
7	1.00	0.33	0.46	1.00	0.54	1.00	0.33	0.90	0.33	0.57
8	0.45	0.58	1.00	0.87	0.42	0.59	0.34	1.00	0.66	0.70
9	0.89	0.36	0.66	0.91	0.33	0.33	0.36	0.77	0.41	0.59

3.4. Calculation of grey relational grade

Calculation of grey relational grade needs the suitable weighting factor for each response. The weighting factor is a very crucial factor because it affects the grading of trials. Hence, in this study, weighting factor has been calculated judiciously and logical manner to avoid any error in the performance calculation. The weights (w_j) based on decision-makers judgement but this must be $\sum w_j = 1$. These weights are decided by the priority matrix as explained in the Section 4.1 and values are as follows, BTE = 0.25, BSFC = 0.16, EGT = 0.02, CP = 0.11, NHR = 0.25, RPR = 0.11, CO = 0.02, HC = 0.02, NOx = 0.02, Smoke = 0.02. After calculating the weights, the grey relational GMdes are calculated using equation 8.

$$\gamma_i = \sum_{j=1}^{n} w_j \omega_{ij} \quad , \text{i=1,2,3,…m} \qquad (8)$$

where γ_i indicates the grey relational GMde for i^ts experiment and w_j is the weighting factor for j^{th} response.

Grey relational grade (GRG) calculation is converting the multi-objective to a single objective in the form of GRG. Using the above equation grey relational grade has been calculated and shown in table 10. For example, calculation of grey relational grade for experiment number 1 is as follows.

0.25*0.33+0.16*1.00+0.02*0.65+0.11*0.35+0.25*0.36+0.11*0.35+0.02*0.54+0.02*0.39+0.02*0.88+0.02*0.67= 0.46

Table 10. Grey relational grade (GRG).

Expt. No.	GRG	Rank
1	0.4582	8
2	0.4206	9
3	0.6056	4
4	0.6599	3
5	0.6604	2
6	0.4870	7
7	0.7117	1
8	0.5185	6
9	0.5569	5

The grey relational grade implies the degree of closeness of comparability sequence to the reference sequence. If the comparability sequence (GRG) value is higher, indicates mores closer to the reference sequence (best) [47]. Therefore, the particular experiment number will be the best choice whose GRG is higher value. The values of GRG from the table 10 indicates that the test number 7 had the highest value as compared to others and ranked 1. Similarly, the ranking of experiment number has been done as per the descending value of GRG as shown in table 10. The operation number 7 is the combination of fuel blend B10, CR 18, and load 100% which gives the best performance characteristics.

3.5 Calculation of optimal factor level effect

In the Taguchi method, performance characteristics are additive. Then it is possible to predict the factor level effect by knowing the main results. In this study, the effect of fuel at level 1 (A_1) can be calculated by averaging the GRG of all experiments where it has level 1. For example, the factor fuel level 1 is B10 and it has in experiment numbers 1, 4, and 7, as shown in table 5 orthogonal array. Therefore, the effect of factor fuel level 1 has been calculated by taking the average value of GRG at these three experiments.

$$A_1 = \frac{1}{3}\left(0.46 + 0.66 + 0.71\right) = 0.61$$

Similarly, all factor levels effect calculated and tabulated in table 11 and ranked the influence of factor on performance characteristics based on the higher value. This ranking of factors effect denotes the load has the highest effect on performance characteristics followed by CR, and fuel. The analysis of variance (ANOVA) for grey relational grade performed to know the further factor's effect.

Table 11. Factor level effects.

Factor	L1	L2	L3	Max-Min	Rank
Fuel	**0.61**	0.53	0.55	0.08	3
CR	0.50	**0.603**	0.60	0.10	2
Load	0.49	0.55	**0.66**	0.17	1

4. Techniques for order preference by similarity to ideal solution (TOPSIS) method

The method of TOPSIS was developed and used by Yoon and Hwang [48] for decision-making problems. The authors have suggested that in the decision-making problems, the preferred alternative should have the shortest distance from the ideal best solution and farthest from the ideal worst solution in terms of geometrical sense. The ideal best solution is a hypothetical term indicates the maximum value of the attribute (response) that gives the optimum solution of the problem and ideal worst solution denotes the minimum value of response in the data base. The TOPSIS method gives the solution that is closest to the ideal best and farthest from the ideal worst solution. It is a multi-criteria decision-making (MCDM) tool, used for selecting best trial/experiment from the available set of tests. It helps to choose the optimal combination of input factors for better performance characteristics in a given trial. The trial/experiment numbers are designed based on the Taguchi's orthogonal array concept for arranging the input factors and their levels in a minimum number of trials. The TOPSIS method was used for accurate weather forecasting application [49]. The optimization of electric discharge machining parameters for the material AISI D2 tool using the TOPSIS method [50]. In this study, TOPSIS was used to rank the experiment numbers as per the performance index values obtained by this method. After arranging the factors and their levels in a nine number of operations, the TOPSIS method used to find the performance index in following steps.

4.1 Normalization of decision matrix (N_{ij})

In this, the experimental results given in the table 5, are normalized by using equation 9 and normalized decision matrix are depicted in table 12. In this method of TOPSIS, alternative means the number of experimental trials and attributes means the output responses.

$$N_{ij} = \frac{m_{ij}}{\left[\sum_{j=1}^{n} m_{ij}^{2}\right]^{\frac{1}{2}}} \tag{9}$$

Table 12. Normalized decision matrix.

Alt.	Attributes									
	BTE	**BSFC**	**EGT**	**CP**	**NHR**	**RPR**	**CO**	**HC**	**Nox**	**Smoke**
1	0.2775	0.3870	0.2960	0.2636	0.2904	0.1782	0.3346	0.4183	0.2567	0.2477
2	0.3190	0.3289	0.3587	0.2530	0.3030	0.1676	0.3309	0.4183	0.3075	0.3223
3	0.3465	0.3289	0.4134	0.2608	0.3658	0.3033	0.6468	0.5878	0.3567	0.5881
4	0.3288	0.3289	0.3313	0.3322	0.3872	0.4348	0.2119	0.2487	0.3150	0.2091
5	0.3403	0.3193	0.3938	0.3382	0.3819	0.4009	0.4238	0.3504	0.3522	0.4356
6	0.3011	0.3580	0.2746	0.3239	0.3263	0.3245	0.2714	0.2261	0.2467	0.1878
7	0.3825	0.2902	0.3435	0.4030	0.3372	0.4518	0.1338	0.1470	0.4487	0.2897
8	0.3155	0.3483	0.2613	0.3892	0.3091	0.3627	0.1413	0.1357	0.2879	0.2408
9	0.3753	0.2999	0.2939	0.3937	0.2801	0.2397	0.1561	0.1696	0.3793	0.2796

4.2. Weight calculation for attributes (responses) (w_j)

The weights are calculated by constructing the priority matrix of responses. In this article, the number of responses is 10, hence, priority matrix 10 × 10 has been prepared. In the priority matrix, the values of judgment made by the authors are entered using the basic scale of relative importance [51]. The verbal judgments 'equally preferred', moderately preferred', strongly preferred', 'very strongly preferred', and extremely preferred are indicated by the numerical values 1,3,5,7, and 9. If the judgments seem to be in between then the numerical values 2, 4, 6, and 8 can be used. The preferences of row over column and vice versa of verbal judgments are tabulated in table 13.

Table 13. Numerical values of verbal judgments.

Equal 1	⇨ **Increasing row importance over column**							
	2	3	4	5	6	7	8	9
	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9
	⇨ **Increasing column importance over row**							

In the priority matrix, the cell entry c_{ij} denotes the numerical value of i^{th} row response compared with j^{th} column response as per the verbal judgments. If i = j, then c_{ij} = 1 and $c_{ji} = 1/c_{ij}$. Hence, in the matrix, all the diagonal entries indicate the response is compared with itself and assigned numerical value 1. The priority matrix has been constructed using the verbal judgments based on their relative importance and tabulated in table 15. Next, multiply the cell values in each row and take the 10^{th} root of multiplication. Then each row's tenth root product is added and is equal to 16.2370. Row 'SUM' denotes the sum of column elements for each response. Weights are calculated for each response by linear normalization of 10^{th} root product. The tenth root of response is divided by the total sum of all responses tenth root product gives the priority vector (weight) for that response. Let us say, weight of response BTE = 3.5887/ 16.2370 = 0.22.

The consistency of values entered in the priority matrix has been checked by consistency index (CI) as given by equation 10.

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{10}$$

where λ_{max} is the total sum of values in the row 'SUM*PV' and is equal to 10.2092, n is the total number of responses, i.e. 10.

CI= 0.023244

After calculation of CI, next to check, how consistently the author has assigned the values in the relative importance of responses. This has been calculated by the consistency ratio (CR) as follows:

CR = CI/RI = 0.023244/1.49 = 0.0156 < 0.1

where RI is the random index, and it depends on the number of responses.
If the value of CR is less than 0.1, then it denotes the decision made by the author is correct, and error is less than 10%, and it is acceptable. The values of RI are given in table 14.

Table 14. Values of RI.

Responses	3	4	5	6	7	8	9	10
RI	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

Table 15. Priority matrix for relative importance of responses.

Responses	BTE	BSFC	EGT	CP	NHR	RPR	CO	HC	NOx	Smoke (HSU)	10^{th} root of product	Priority vector (PV)= Weights
BTE	1	2	9	3	1	3	9	9	9	9	4.0054	0.25
BSFC	1/2	1	7	2	1/2	2	7	7	7	7	2.6458	0.16
EGT	1/9	1/7	1	1/6	1/9	1/6	1	1	1	1	0.3707	0.02
CP	1/3	1/2	6	1	1/3	1	6	6	6	6	1.8346	0.11
NHR	1	2	9	3	1	3	9	9	9	9	4.0054	0.25
RPR	1/3	1/2	6	1	1/3	1	6	6	6	6	1.8346	0.11
CO	1/9	1/7	1	1/6	1/9	1/6	1	1	1	1	0.3707	0.02
HC	1/9	1/7	1	1/6	1/9	1/6	1	1	1	1	0.3707	0.02
NOx	1/9	1/7	1	1/6	1/9	1/6	1	1	1	1	0.3707	0.02
Smoke(HSU)	1/9	1/7	1	1/6	1/9	1/6	1	1	1	1	0.3707	0.02
SUM	3.7222	6.7143	42	10.8333	3.7222	10.8333	42	42	42	42	16.1793	1
SUM*PV	0.9215	1.0980	0.9623	1.2284	0.9215	1.2284	0.9623	0.9623	0.9623	0.9623	10.2092	

Using the priority matrix, the calculated weights for each of the responses are as follows: BTE = 0.25, BSFC = 0.16, EGT = 0.02, CP = 0.11, NHR = 0.25, RPR = 0.11, CO = 0.02, HC = 0.02, NOx = 0.02, Smoke = 0.02.

4.3 Calculation of weighted normalized matrix (V_{ij})

After calculation of normalized decision matrix and weights for all the attributes, the weighted normalized matrix is calculated using the following equation and tabulated in table 15.

$$V_{ij} = w_j N_{ij}$$

Table 16. Weighted normalized matrix.

Alt.	Attributes									
	BTE	BSFC	EGT	CP	NHR	RPR	CO	HC	Nox	Smoke
1	0.0610	0.0890	0.0030	0.0422	0.0523	0.0160	0.0067	0.0125	0.0077	0.0074
2	0.0702	0.0757	0.0036	0.0405	0.0545	0.0151	0.0066	0.0125	0.0092	0.0097
3	0.0762	0.0757	0.0041	0.0417	0.0658	0.0273	0.0129	0.0176	0.0107	0.0176
4	0.0723	0.0757	0.0033	0.0531	0.0697	0.0391	0.0042	0.0075	0.0095	0.0063
5	0.0749	0.0734	0.0039	0.0541	0.0688	0.0361	0.0085	0.0105	0.0106	0.0131
6	0.0662	0.0823	0.0027	0.0518	0.0587	0.0292	0.0054	0.0068	0.0074	0.0056
7	0.0841	0.0668	0.0034	0.0645	0.0607	0.0407	0.0027	0.0044	0.0135	0.0087
8	0.0694	0.0801	0.0026	0.0623	0.0556	0.0326	0.0028	0.0041	0.0086	0.0072
9	0.0826	0.0690	0.0029	0.0630	0.0504	0.0216	0.0031	0.0051	0.0114	0.0084

Next step is to calculate the ideal best an ideal worst solution using the following equations:

$$V^{+} = \max_j V_{ij}\,(j \in J), \min_j V_{ij}\,(j \in J')$$
$$V^{-} = \min_j V_{ij}\,(j \in J), \max_j V_{ij}\,(j \in J')$$

where I = 1, 2,…..m indicates the alternatives and j = 1, 2, . . . n are the attributes.
The term J used for beneficial term, i.e. larger the better and J' used for non-beneficial terms (smaller the better). Using these equations and conventions the ideal best and ideal worst is calculated for each attributes and tabulated in the table 17.

Table 17. Calculated ideal best (V^+) and ideal worst (V^-).

	BTE	BSFC	EGT	CP	NHR	RPR	CO	HC	NOx	Smoke
V^+	0.0841	0.0668	0.0026	0.0645	0.0697	0.0407	0.0027	0.0041	0.0074	0.0056
V^-	0.0610	0.0890	0.0041	0.0405	0.0504	0.0151	0.0129	0.0176	0.0135	0.0176

4.4 Calculation of separation variables

The separation variable indicates its Euclidean distance from the ideal solutions for each alternative. These variables are calculated using the below given equations.

$$S_i^{+} = \left[\sum_{j=1}^{n}\left(V_{ij} - V_j^{+}\right)^2\right]^{0.5}$$

$$S_i^{-} = \left[\sum_{j=1}^{n}\left(V_{ij} - V_j^{-}\right)^2\right]^{0.5}$$

Table 18 shows the calculated separation variables for each alternative.

Table 18. Separation variables.

Alt.	S^+	S^-
1	0.050258	0.014477
2	0.042929	0.020634
3	0.036049	0.028407
4	0.019162	0.041707
5	0.02004	0.038673
6	0.031437	0.028697
7	0.011324	0.052182
8	0.025779	0.03711
9	0.027789	0.041907

The relative closeness P_i of each alternative has been calculated using the following equation and tabulated in table 19. Based on the value of relative closeness, the alternatives are ranked in descending order for the value of P_i .

$$P_i = \frac{S_i^{-}}{\left(S_i^{-} + S_i^{+}\right)}$$

Table 19. Relative closeness of alternatives.

Alt.	P_i	Rank
1	0.22364	9
2	0.324625	8
3	0.440723	7
4	0.685191	2
5	0.658678	3
6	0.477218	6
7	0.821683	1
8	0.590088	5
9	0.601279	4

Finally, the ranking of TOPSIS alternatives has been compared with GTM and depicted in Table 20. It is found that the optimal solution is same in the both the technique and confirms the results.

Table 20. Ranking comparison of GTM and TOPSIS.

Expt. No.	1	2	3	4	5	6	7	8	9
GTM	8	9	4	3	2	7	1	6	5
TOPSIS	9	8	7	2	3	6	1	5	4

5. Results and Discussion

5.1 Signal to noise ratio analysis for GRG

The Taguchi method is used for analysis of grey relation grade obtained by GM the optimization technique. The signal to noise ratio of GRG calculated by using the 'larger the better' design condition represented by equation 3. The S/N ratio of input factors and their levels are given table 21. Similarly, figure 4 shows the graphical representation of S/N ratio of GRG for input elements. The inferences have drawn from table 12 and figure 4, about optimal combination of input factors $A_1B_2C_3$, i.e. fuel blend B10, CR 16, and load 100% for improved performance characteristics.

Table 21. Signal to noise ratios for GRG.

Level	Fuel	CR	Load
1	-4.443	-6.191	-6.207
2	-5.608	-4.471	-5.393
3	-5.175	-4.564	-3.626
Delta	1.165	1.72	2.581
Rank	3	2	1

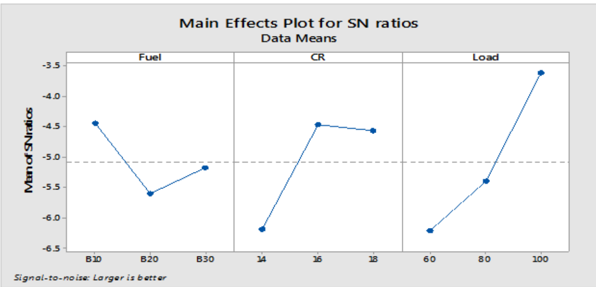

Figure 4. S/N ratio for grey relational GMde (GRG)

5.2 Analysis of variance

For analyzing the effect of input factors on output responses quantitatively, the statistical tool analysis of variance (ANOVA) has been used at 95% confidence level. The ANOVA distributes the total variability in responses, among the available factors. This investigation recognizes the factor whose effect is significant on the output responses and quantifying the effect of the factor regarding percentage contribution [52,53]. The various terms used for analysis of variance namely sum of squares (SS), mean square (MS), and F-value [54] calculated by using the below given equations, and the results are depicted in table 22.

$$SS = \sum_{i=1}^{n}\left(\gamma_i - \gamma_m\right)^2$$

$$MS = \frac{SS}{DF}$$

where γ_i is the GRG for ith experiment and γ_m is the mean of GRG. The value of F is calculated by taking the ratio of factor MS to the error mean square. The percentage contribution is the ratio factor SS to total SS.

The analysis of variance of grey relational grade has been performed, and the results depict in table 18. The ANOVA result shows the most significant factor is load whose contribution for variation of output responses is 52.82%, followed by the CR (28.38%), and fuel (10.52%). This result implies that by controlling the load and CR, the performance parameters can be varied and improved.

Table 22. Analysis of variance for S/N ratios.

Source	DF	Seq SS	Adj MS	F	Contribution (%)
Fuel	2	2.081	1.0405	1.27	10.52
CR	2	5.614	2.8071	3.42	28.38
Load	2	10.449	5.2244	6.37	52.82
Residual Error	2	1.639	0.8197		8.28
Total	8	19.784			100

5.3 Experiments for confirmation test

The initial optimal settings of variable compression ratio diesel suggested by the manufacture are fuel petrodiesel, CR 17.5, and load 80. Then at initial settings of engine, the GRG (0.521) and S/N ratio (- 3.4525) are represented in table 14 and compared with predicted and experimental optimized factor level results.

After getting, the optimal input factor levels from the S/N ratio plots, the grey relational GMde ψ is predicted at the optimal input factor by Equation 10.

$$\psi = \gamma_m + \sum_{i=1}^{n}\left(\gamma_i - \gamma_m\right) \qquad (10)$$

Using equation 10, the predicted grey relational grade at optimal input factor level $A_1B_2C_3$ is 0.8429 and corresponding S/N ratio is -1.4845 tabulated in table 14. Also at the same optimal level, confirmation experiment has been conducted and the results are reproduced. From the results, it is inferred that the improvement in GRG with linseed biodiesel blend is 0.2922, i.e. 56.1% as compared to the existing initial setting of variable compression ratio diesel engine. The optimal combination of input parameters shows the improvement in performance, combustion, and emission parameters justifying the application of Taguchi with GM.

Table 23. Results of existing and optimal setting of input factors.

	Initial process parameters	Optimal process parameters	
	Existing VCR engine setting	Predicted	Experimental
Level		$A_1B_2B_3$	$A_1B_2C_3$
GRG	0.521	0.8429	0.8132
S/N ratio	-3.4525	-1.4845	-2.38936

Improvement in GRG = (0.8132-0.521)= 0.2922

5.4 Performance of index (p_i) of TOPSIS

Taguchi's OA, i.e. arrangement of factors and levels in minimum number of experiment taken as the alternatives and corresponding set of output parameters as attributes (responses). This OA has been analyzed using TOPSIS for selecting the best option. The performance index p_i for TOPSIS calculated and it show the best alternative is 7. This 7th choice has a combination of B10, CR18, and load 100% and this optimum result exactly matching with GTM.

6. Conclusion

In the variable compression ratio diesel engine, the performance, combustion, and emission characteristics are affected by the input factor fuel blends, compression ratio, and applied load on the engine. In the present study to obtain the defined objective the Taguchi method of analysis was integrated with grey relational analysis and analytical hierarchy process. The hybrid technique GTM used to identify the best combination of input factor levels for improved performance characteristics in minimum number of trials. In designing of experiment the GTM method is useful, since it reduces the time to perform the experiments by suggesting the optimal combination of factors in lesser number of tests. The following conclusions were drawn:

1. In the study, the optimal combination are fuel blend B10, compression ratio 18 and load on the engine is 100% for better performance and combustion and reduced emission. The optimum factor level effects observed by these techniques are fuel blend B10 and compression ratio 16 and load 100%.

2. The ANOVA analysis for grey relational grade shows the most influencing factor is load with contribution of 52.28% followed by the factor CR of 28.38% and least effect of fuel blend of contribution only 10.52%.

3. The confirmation test shows the improvement in performance by 56.1% as compared to the existing setting of engine for fuel as petro diesel. This confirms the advantage of GTM techniques for optimization.

4. The GTM results also confirmed by Taguchi-TOPSIS for the optimal level of the factor. This means hybrid method of optimization can be used effectively.

7. Future suggestions

Based on this experimental and parametric optimization study, the existing engine's feasibility can be studied further for other available vegetable oil based alternative fuels.

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9. Nomenclature and abbreviations

ANOVA	Analysis of variance
BSFC	Break specific fuel consumption
bTDC	Before top dead centre
BTE,	Break thermal efficiency
B-XX (e.g.B10)	B-biodiesel blend, XX- percentage of biodiesel
CI	Consistency index
CNG	Compressed natural gas
CO	Carbon monoxide
CP	Combustion pressure
CR	Compression ratio / consistency ratio
DF	Degree of freedom
DOE	Design of experiments
EGR	Exhaust gas recirculation
EGT	Exhaust gas temperature
GRG	Grey relational grade
GTM	Grey Taguchi method
HC	Hydrocarbon
IP	Injection pressure
LPG	Liquid petroleum gas
MS	Mean square
NHC	Net hydrocarbons
NOx	Nitrogen oxide
PSO	particle swarm optimization
PV	Priority vector
RI	Random index
RPR	Rate of pressure rise
S/N	Signal to noise ratio
SS	Sum of square
TOPSIS	Techniques for order preference by similarity to ideal solution
VCR	Variable compression ratio

Greek symbols

m_{ijk}	Simulation result for trial i for response j , in k^{th} number of replication
S^+, S^-	Separation variables
V^+, V^-	Ideal best and ideal worst
V_{ij}	Weighted normalized matrix
w_j	Weighting factor
γ_i	Grey relational grade
ζ	Distinguishing coefficient
η_{ij}	S/N ratio of experiment number i for response j
ω_{ij}	Grey relational generation
ψ	Grey relational GMde

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