Optimized Tilted Solar Radiation in Equator Region: Case Study of Seven Climatic Zones in Tanzania

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Abstract
This study delves into the ongoing discourse surrounding the optimal tilt angles for solar panels to maximize solar PV power generation. Focused on seven equatorial regions in Tanzania; Dodoma, Dar es Salaam, Kilimanjaro, Kigoma, Iringa, Mtwara, and Mwanza. Multiple mathematical models are employed to ascertain the most efficient panel tilts. Leveraging solar radiation data spanning from 2000 to 2017, we developed an algorithm, specifically tailored for computing suitable tilt angles in the southern hemisphere. Our investigation reveals compelling insights into the variation of optimal panel tilts throughout the year. Notably, the monthly optimal tilt angles fluctuate significantly across the regions. June emerges as the month with the highest recorded monthly optimal tilt angle, ranging from 45 degrees in Mtwara to 31 degrees in Kilimanjaro. Conversely, December showcases the lowest tilt angles, spanning from -30 degrees in Mwanza to -26 degrees in both Kigoma and Iringa. Quarterly angles exhibit peaks during the second quarter of the year, reaching 39 degrees in Mtwara and 27 degrees in Kilimanjaro, while experiencing declines in the fourth quarter, plunging to levels between -19 and -24 degrees. Additionally, our study calculates annual optimal tilt angles, revealing a range from 2 degrees in Kilimanjaro to 11 degrees in Mtwara. Crucially, the deployment of monthly optimally tilted solar PV panels demonstrates a noteworthy enhancement, yielding a 6-11% gain in solar radiation compared to horizontally mounted panels. Our study advocates for the adoption of dynamic tilt adjustment strategies of periodic angle alterations to maximize solar PV power generation.

Keywords: *Optimal tilt angles, Solar radiation, Solar panels, Equator region.*

1. Introduction
Optimizing the tilt angle of solar panels stands as a pivotal technique in enhancing the efficiency of PV-based energy generation systems. The effectiveness of photovoltaic (PV) units heavily relies on the solar radiation incident upon the panels [1]. The magnitude of solar radiation reaching these panels hinges upon two crucial factors: the direction and tilt angle of the solar panels. The ideal tilt angle varies dynamically based on the sun's position relative to the Earth, changing on a daily, monthly, and annual basis. Moreover, this optimal angle is location-dependent, influenced by the geographical latitude, prevailing climate conditions, solar radiation patterns, and the intended duration of utilization [1]. Ensuring the panels are set at an optimal tilt angle throughout the year becomes imperative to maximize energy generation efficiency.

Different researchers have used different approaches varying from pre-determined mathematical models [2, 3] to complex computer algorithms [4, 5]. Several models have been proposed for calculating the tilted to horizontal diffuse radiation ratio R_d; they are classified as isotropic [6-9] and anisotropic [10-13]. Both approaches require the choice of the tilted to horizontal diffuse radiation ratio (R_d) to be calculated by either an isotropic or anisotropic models. The isotropic models assume that the power of diffuse sky radiation is uniform over the sky vault; subsequently, the diffuse radiation occurrence on a tilted surface relies upon the part of the sky arch seen by it. The anisotropic models accept the anisotropy of the diffuse sky radiation

in the circumsolar area (sky close to the sun-based plate) and isotropically appropriated diffuse part of the remainder of the sky vault [2]. The isotropic models are based on simple calculations, but mostly, the results do not give adequate accuracy [3, 5]. Anisotropic models are proposed to compute the diffuse sun radiation on tilted surfaces precisely. However, these models are more complex than the isotropic ones in math estimation and PC time prerequisites [5, 10].

Several other studies including Jacobson and Jadhav [14], Ozbay, Karafil [15], Kaddoura, Ramli [16], and Salih [17] employ manual or computational approaches to determine tilt angles, utilizing tools like the PV watts calculator, Raspberry Pi microcontroller, Solar Panel Angle Calculator program, and MATLAB programming. These methodologies involve either setting panels at varying angles, using solar radiation data, or formulating models based on meteorological data. Others, such as Al, Karamallah [18], Nfaoui and El-Hami [19], Salari and Jahanshahi Javaran [20], and Buzra, Mitrushi [21] employ software-based approaches using tools like Engineering Equations Solver (EES) software and Bernard-Menguy-Schwartz (BMS) model to estimate solar radiation and determine optimal tilt angles. These studies rely on data sources such as NASA, Iranian Meteorological Organization, METEONORM, and local meteorological station data.

Furthermore, studies like Ullah, Imran [22], Dutta, Biswas [23], Fung and Xing [24], and Aguinsatan and Remedio [25] utilize a combination of mathematical models, computer programming, and on-site measurements to simulate solar radiation and optimize tilt angles. Additionally, methodologies such as Shu, Kameda [26] and Le Roux [3] involve installing PV modules at varying tilt angles and deducing optimal angles from experimental data.

The literature survey indicates a predominant reliance on software for simulating solar radiation models, with limited studies leveraging hardware setups in solar PV systems to ascertain optimal angles [27]. Overall, the studies employ a variety of techniques including manual, computational, software-based, and experimental approaches, using a range of software tools and data sources to determine the optimal tilt angles for PV systems across diverse global locations.

Tanzania, situated near the equator, possesses significant solar energy potential [28]. Optimizing tilt angles for solar PV systems in Tanzania aims to maximize energy capture and enhance the efficiency of solar power generation. By fine-tuning the tilt angles according to local solar radiation patterns and climatic conditions, it becomes possible to harness more solar energy throughout the year. While studies worldwide have explored tilt angle optimization, there's a research gap in Tanzania-specific data. Localized research accounting for Tanzania's unique geographical coordinates, solar radiation characteristics, and climatic nuances would provide more accurate and relevant optimal tilt angle recommendations.

2. Methodology

2.1. Study site

The study utilized solar irradiance data from the Tanzanian Meteorological Agency (TMA) for three climatological zones, while for the remaining four zones; satellite irradiance data from The National Aeronautics and Space Administration (NASA) were utilized. Table 1 summarizes the seven representative Climatological zones along with their corresponding data sources including latitude, longitude, elevation, and data sources [29].

The climatological zone division is based on the natural climatology of Tanzania general vegetation and relief. The division is such that areas with the same natural climatic condition are grouped together, as detailed elsewhere [30].

Table 1. Study sites and data sources.

Zone	Region	Latitude (degrees)	Longitude (degrees)	Elevation (m)	Data source
Central	Dodoma	6.16 S	35.75 E	1120	TMA
Northern Coast	Dar es Salaam	6.82 S	39.27 E	24	TMA
North Eastern Highland	Kilimanjaro	3.07 S	37.36 E	1800	TMA
Western Zone	Kigoma	4.53 S	29.48 E	885	NASA
Southern Coast	Mtwara	10.31 S	40.18 E	113	NASA
Southern Western Highland	Iringa	7.77 S	35.69 E	1640	NASA
Lake Victoria Basin	Mwanza	2.52 S	32.92 E	1140	NASA

2.2. Solar radiation modelling

Meteorological data generally provide global solar irradiance on horizontal surfaces, necessitating correlation methods to estimate solar insolation on tilted surfaces. In this study, selected mathematical models were interconnected, where

the output of one model serves as input to another. Monthly average daily global tilted radiation was computed using 17 years of horizontal radiation data, establishing monthly averages from 2000 to 2017.

Monthly average daily total radiation on a tilted surface (H_T) is estimated by considering the direct beam (H_B), diffuses (H_D), and reflected components (H_R) of radiation on the surface. The incident total radiation (H_T) is derived from Equations (1)-(4). These equations represent the interrelationship between various radiation components (H_B, H_D, H_R) and their relationships with global and diffuse radiation on horizontal surfaces. The calculation involves factors such as surface tilt, latitude, declination angle, and sunset hour angle [2].

$$H_T = H_B + H_D + H_R \tag{1}$$

The H_B, H_D and H_R can be calculated from:

$$H_B = (H_g - H_d)R_b \tag{2}$$

$$H_D = H_d R_d \tag{3}$$

$$H_R = H_g \rho R_r \tag{4}$$

where H_g and H_d are the global and diffuse radiation on a horizontal surface, and R_b, Rd, and R_r are the ratios of tilted to horizontal beam, diffuse and reflected radiation, respectively. The magnitude ρ in Equation (4) is the effective ground reflectance, which varies from 0.04 to 0.75 depending on the reflecting surface [31]. Tanzania, being a tropical country, does not get snow cover, and the ground is covered by both green leaves ($\rho = 0.26$) and dry grass ($\rho = 0.2$) [3]. An average of effective ground reflectance between green leaves and dry grass was then calculated, and a value of $\rho = 0.23$ was used in this study. For the southern hemisphere, the ratio R_b can be calculated as advised by [31];

$$R_b = Cos(\phi + \beta)Cos\delta Sin\omega_{ss} + \omega_{ss}Sin(\phi + \beta)Sin\delta / Cos\phi Cos\delta Sin\omega_{ss} + \omega_{ss}Sin\phi Sin\delta \tag{5}$$

ϕ, β, δ, and w_{ss} are latitude of the site, tilt angle of the solar collector, declination angle, and sunset hour angle, respectively. The declination of the sun was calculated as advised by [31]:

$$\delta = 23.45 sin\left(\frac{360}{365}(n + 284)\right) \tag{6}$$

and the sunset hour angle can be computed as follows [2]:

$$\omega_{ss} = Arccos(-\tan(\phi)\tan(\delta)) \tag{7}$$

The anisotropic model developed by [13] was adopted in this study to determine the ratio of diffuse radiation on tilted surfaces (R_d).

$$R_d = \frac{H_b}{H_o}R_b + \left(1 - \frac{H_b}{H_o}\right)\left(1 + \frac{cos\beta}{2}\right)\left(1 + \sqrt{\frac{H_b}{H_g}}Sin^3\left(\frac{\beta}{2}\right)\right) \tag{8}$$

The extraterrestrial solar radiation was calculated using a method by [31];

$$H_o = I_{SC}\frac{24*3600}{\pi}\left(1 + 0.033cos\frac{360n}{365}\right) * \left(\frac{\pi\omega_{ss}}{180}sin\phi sin\delta + cos\phi cos\delta cos\omega_{ss}\right) \tag{9}$$

where I_{SC} is a solar constant, and for this work, the calculations were based on 1366.1 W/m². To estimate the monthly average daily diffuse irradiation on horizontal surfaces (H_d), the model of [32] was applied:

$$\frac{H_d}{H_g} = \{1.391 - 3.560K_T + 4.189K_T^2 - 2.137K_T^3 \text{ if } \omega_{ss} \leq 81.4\} \tag{10}$$

$$\frac{H_d}{H_g} = \{1.311 - 3.022K_T + 3.427K_T^2 - 1.821K_T^3 \text{ if } \omega_{ss} \geq 81.4\} \tag{11}$$

$$K_T = \frac{H_g}{H_o} \tag{12}$$

Equations (5)-(12) establish the relationship between the different radiation components and their ratios, considering variables such as tilt angle, declination, and extraterrestrial radiation. Beam horizontal radiation (H_b) was then computed as:

$$H_b = H_g - H_d \tag{13}$$

The ratio R_r was computed as detailed by [4]:

$$R_r = \frac{1 - cos\beta}{2} \tag{14}$$

2.3. Matlab script

In order to optimize the monthly tilt angles for solar PV in the southern hemisphere near the equator, a script was developed in the MATLAB live editor. This script utilized Equations 1 to 14, and employed MATLAB element-wise matrix operations. By doing so, the script was able to calculate the values of global tilted radiation using horizontally measured radiation data for various tilt angles within a predetermined range.

To determine the optimal solar panel tilt angle for each month, tilted global radiation was computed using angles ranging from -35 to 90 degrees in 1-degree increments. The tilt angle that received the highest amount of global tilted monthly average

daily solar radiation was selected as the optimal angle.

The MATLAB algorithm was designed so that only one variable needed to be changed each month for each climatological region. This variable was the Julian day of the year, which was adjusted according to the schedule provided in table 2.

The computations were performed for each region from January to December. The results were then sorted in a Microsoft Excel spreadsheet based on the largest tilted global radiation value to the smallest. The tilt angles associated with the highest tilted radiation values were chosen as the optimum tilt angles for each month. The selection of this optimization approach was based on its simplicity and replicability. The MATLAB script used for monthly and quarterly tilt angle optimization is provided as appendix 1.

Table 2: Representative averaged days for months[1].

ith day of the month	Month	n for ith day of the month	Julian day of the year (n)
17	January	i	17
16	February	31 + i	47
16	March	59 + i	75
15	April	90 + i	105
15	May	120 + i	135
11	June	151 + i	162
17	July	181 + i	198
16	August	212 + i	228
15	September	243 + i	258
15	October	273 + i	288
14	November	304 + i	318
10	December	334 + i	344

While other researchers [2, 34] have used seasonally adjusted tilt angles to account for the four seasons of the year, in this study, Tanzania's proximity to the equator in the southern hemisphere led to a different approach. Instead of considering the traditional four seasons, the year was divided into four quarters of three months each. The first quarter included January to March, the second quarter included April to June, the third quarter included July to September, and the fourth quarter included October to December. For the quarterly analysis, the same MATLAB script used for the monthly computations was employed. However, instead of calculating the global tilted radiation for each individual month, an average of the global horizontal radiation was calculated for each quarter.

[1] 33.Duffie, J.A., W.A. Beckman, and N. Blair, Solar Engineering of Thermal Processes, Photovoltaics and Wind. 2020: John Wiley & Sons.

This average was then used to compute the tilted global radiation. The representative days for each quarter (nquarter) were determined by averaging the representative days for each month (n) within that quarter. The same range of tilt angles as used in the monthly computations was applied for the quarterly analysis. The tilt angle that resulted in the highest amount of radiation was selected as the optimal angle for that quarter. Similarly, to calculate the yearly optimum tilt angle, the monthly optimal tilt angles were averaged [2]. This approach was adopted for each climatological region under consideration.

3. Results and discussion

3.1. Monthly optimal tilt angles

The computed monthly optimal tilt angles for the seven climatological regions are visually presented in figures 1 and 2. Across these regions, a consistent trend emerges, where optimal monthly tilt angles progressively rise from January, reaching their apex in June, and subsequently decline, hitting their lowest points in December. This variance suggests a significant correlation between solar panel tilt angles and the changing position of the sun throughout the year.
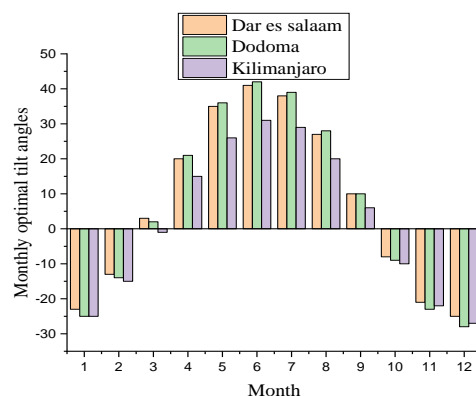

Figure 1. Monthly optimal tilt angles for three regions whose solar radiation data was measured from the ground.
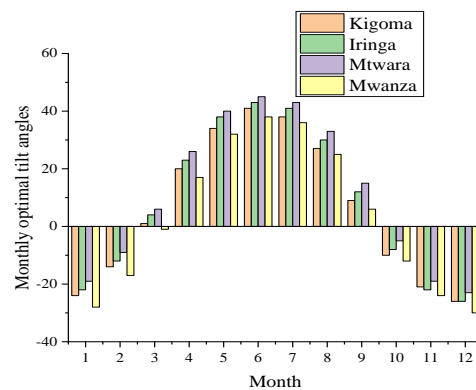

Figure 2. Monthly optimal tilt angles for four regions whose solar radiation data was measured by NASA satellite.

During the Tanzanian summer months (December to February), lower tilt angles appear optimal due to the sun's elevated position in the sky, ensuring maximum sunlight exposure. Conversely, in the winter months (June to August), higher tilt angles tend to optimize energy capture due to the sun's lower position in the sky. This seasonal adaptation of tilt angles aligns with the Earth's tilt relative to the sun; in June, the southern hemisphere is tilted farthest from the sun, whereas in December, it is tilted closest. The range of optimal monthly tilt angles exhibits remarkable variation, with the Mwanza region recording the lowest optimal tilt angle of $-30°$ in December, while the Mtwara region shows the highest tilt angle of $45°$ in June.

3.2. Quarterly and annual optimal tilt angles

Quarterly optimal tilt angles demonstrate a consistent pattern across all regions, showcasing a direct correlation between the changing seasons and tilt angle variations. Figure 3 illustrates this trend, depicting lower tilt angles during the first and fourth quarters and higher angles during the second and third quarters.

Specifically, the first quarter displays lower optimal tilt angles, ranging from $-17°$ in Mwanza to $-9°$ in Mtwara regions. In contrast, the second and third quarters show higher optimal angles, ranging from $27°$ in Kilimanjaro to $39°$ in Mwanza (Q2) and from $20°$ in Kilimanjaro to $33°$ in Mtwara (Q3). The fourth quarter reflects a return to lower optimal tilt angles, varying from $-24°$ in Mwanza to $-19°$ in Mtwara.
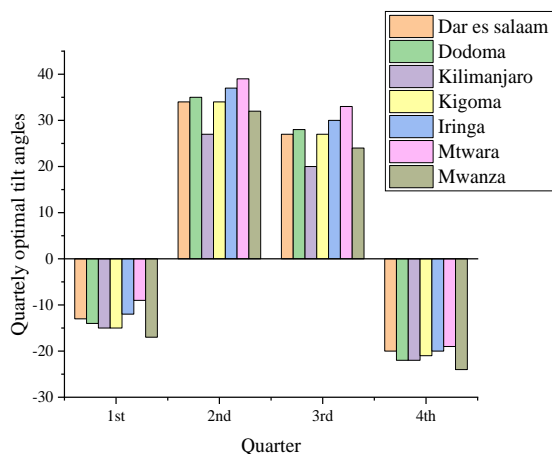

Figure 3. Quarterly optimal tilt angles in seven climatological zones.

Overall, the annual optimal tilt angles show an incremental increase corresponding to higher latitudes and closely mirror the latitudes of the seven regions (Figure 4). This corroborates the findings of various researchers [2, 3, 14, 35, 36], emphasizing the dependence of annual optimal tilt angles on site latitude.

Optimal tilt angles for solar panels are crucial because they significantly impact the energy output and overall efficiency of photovoltaic (PV) systems [23]. This study's insights into monthly, quarterly and annual optimal tilt angles for solar panels hold substantial promise for enhancing energy capture, reducing costs, and promoting sustainable energy practices. Future research avenues focusing on climate change impacts, seasonal adaptations, cost and advanced modelling techniques will further refine and expand practical applications of this knowledge. These findings bear significance in advancing renewable energy adoption and sustainable development initiatives on both practical and managerial fronts in sub Saharan Africa.
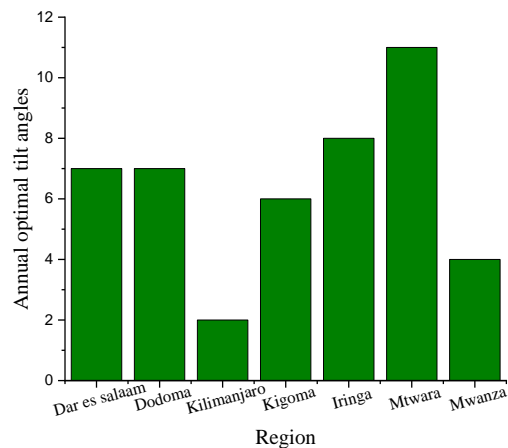

Figure 4. Annual optimal tilt angles in seven climatological zones.

3.3. Comparison of monthly averaged daily solar radiation values at different tilt angles

Comparison of annual gain in solar radiation was done between monthly optimal tilt, quarterly optimal tilt, fixed annual optimum tilt, and fixed annual vertical tilt against the horizontal tilt for the seven regions as shown in table 3.

$$\textbf{Gain or Loss } (\%) = \left(\frac{\beta_i}{\beta_0} - 1\right) * 100 \qquad (15)$$

where the subscripts i correspond to monthly, quarterly, annual or vertical tilt angles for solar panels.

All seven regions received the highest amount of solar radiation when the tilt angles are adjusted monthly. The highest gain was in Mtwara region (11%), and the lowest in Kilimanjaro (5%).

Table 3. Annual solar insolation (in MJ/m^2-year) as received by a PV panel at different tilt angle adjustments.

Region	β₁ (Monthly optimal)	β₂ (Quarterly optimal)	β₃ (Annual optimal)	β₄ (Vertical 90°)	β5 (Horizontal 0°)
Dar es Salaam	7128	7100	6647	2481	6623
Gain/Loss (%)	8	7	0	-63	0
Dodoma	8753	8738	8061	2683	8009
Gain/Loss (%)	9	9	1	-66	0
Kilimanjaro	5568	5499	5246	1760	5246
Gain/Loss (%)	6	5	0	-66	0
Kigoma	7880	7853	7319	2969	7241
Gain/Loss (%)	9	8	1	-59	0
Iringa	9017	8745	8303	3080	8191
Gain/Loss (%)	10	7	1	-62	0
Mtwara	8370	8398	7729	3263	7568
Gain/Loss (%)	11	11	2	-57	0
Mwanza	8486	8237	7837	2479	7812
Gain/Loss (%)	9	5	0	-68	0

Quarterly optimal tilt angles have lower gain against the horizontal for Dar es Salaam, Kilimanjaro, Iringa, and Mwanza regions when compared to the monthly optimal tilt angles. However, there is still a significant gain against the horizontal in all seven regions, as seen in table 3, as such when changing the tilt angle on a monthly basis is not practical; then the tilt angles can be changed in three-month intervals to optimize the solar radiation gain.

All seven regions are predicted to have small or negligible gains within the range of 0% to 2% for the annual fixed tilt angle compared to the horizontal maintenance: This can be attributed to the fact that when the sun's rays strike Earth's surface near the equator, the incoming solar radiation is more direct (nearly perpendicular or closer to a 90˚ angle). In turn, this favors lower tilt angles for solar collectors at lower latitude angles. At higher latitudes, the angle of solar radiation is small, causing the energy to be spread over a larger area of the surface [29]. A vertical tilt, however, has shown a considerable loss in solar radiation when compared to the horizontal. The loss is in the range of 68% in the Mwanza region to 62% in Iringa region. As such, building integrated solar PV (BIPV) application in all seven climatological zones can only be viable when the solar cells are integrated within the roof of the buildings; vertical walls may not be suitable for BIPV.

4. Conclusion

Our comprehensive investigation into solar PV tilt angles across seven distinct climatological zones in Tanzania has yielded crucial insights and optimization strategies. Our findings demonstrate that monthly and quarterly adjustments of tilt angles for solar panels lead to substantial gains in solar radiation when compared to a fixed yearly tilt angle. This nuanced approach of adjusting tilt angles at shorter intervals significantly enhances electric power production from solar PV systems.

The observed increase in energy gain and efficiency proves particularly advantageous for medium to large-scale solar installations. These optimizations directly impact the cost-effectiveness of establishment and production per kWh, offering a compelling proposition for enhancing the economic viability of solar projects. Our study also highlights an intriguing trend: the optimal annual fixed tilt angle closely aligns with the latitude angle for each region. However, in regions with lower latitude angles such as Kilimanjaro and Mwanza, an intriguing possibility arises. Solar panels in these areas could potentially remain fixed horizontally throughout the year if the logistical feasibility of adjusting tilt angles monthly or quarterly poses challenges.

Given these discoveries, we advocate for site-specific investigations to precisely determine the optimal angles before mounting solar panels. Tailoring the tilt angles according to the unique solar radiation characteristics of each location can unlock untapped potential for maximizing energy capture and bolstering the efficiency of solar PV systems.

Further research focusing on site-specific studies and logistical feasibility assessments would be beneficial. Investigating the practicality and economic implications of implementing shorter interval adjustments against fixed horizontal angles in specific regions could provide valuable insights, particularly for areas where frequent angle alterations might be challenging.

This proactive approach can pave the way for more informed decisions in solar energy deployment, and pave the path towards greater efficiency and cost-effectiveness in harnessing solar power in Tanzania."

5. Appendix 1 (MATLAB live editor script developed and used for tilt angle optimization.)

```
%Data and input section
L = Input here the latitude angle of the site or region;
n = Input the representative day for a month or a
quarter;
Isc = 1366.1;
Hg = Input here global horizontal radiation;
%Representative days for each month n =
[17,47,75,105,135,162,198,228,258,288,318,344]
% Tilt angle set
T = [Input here any range of tilt angles for a solar
collector, separate the values to tabs]

% Declination angle formula
d=23.45*sind(360/365*(n+284));
%Ratio HB/Hb
hs1 = acosd(-tand(L)*tand(d));
Rb =
(cosd(L+T)*cosd(d)*sind(hs1)+(pi/180)*hs1*sind(L+T
)*sind(d))/(cosd(L)*cosd(d)*sind(hs1)+(pi/180)*hs1*si
nd(L)*sind(d));
%Ratio HR/Hr
Rr=(1-cosd(T))/2;
%Extraterrestrial radiation
Ho=
(Isc*24*3600)/pi*(1+0.033*cosd(360*n/365))*(pi*hs1
/180*sind(L)*sind(d)+cosd(L)*cosd(d)*sind(hs1));
%Ratio Hg/Ho
KT = Hg/Ho;
if hs1<=81.4
    Hdg = 1.391-3.560*KT+4.189*KT.^2-2.137*KT.^3;
elseif  hs1>81.4
    Hdg = 1.311-3.022*KT+3.427*KT.^2-1.821*KT.^3;
end
Hd = Hdg*Hg;
Hb = Hg-Hd;
HB = (Hg-Hd)*Rb;
HR = Hg*0.23*Rr;

%Ratio HD/Hd
%Rd = a*Rb+(1-
a)*(1+f*sind(T/2).^3)*((1+cosd(T))/2);
a = Hb/Ho;
f = sqrt(Hb/Hg);
y = a*Rb;
k = (1-a)*(1+f.*sind(T/2).^3);
j = ((1+cosd(T))/2);
Rd = y+k.*j;

HD = Hd*Rd;
HT = HB + HD + HR;
```

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