

Assessment of Progress and Regression of Iran's Regional Electric Companies: Extended Network SBM Model

H. Seyed Zaman^{1*}, F. Hossinzadeh Lotfi² and M. Ahadzadeh Namin³

1. Faculty of Engineering, Islamic Azad University, Central Tehran Branch, Tehran, Iran.

2. Department of Basic Science, Science and Research Branch, Islamic Azad University, Tehran, Iran.

3. Department of Mathematics, Shahr-e-Qods Branch, Islamic Azad University, Tehran, Iran.

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*Corresponding author: s.z.hosseini@tavanir.org.ir (H. Seyed Zaman)

Abstract

Assessment of progress and regression in the performance of organizations is necessary to determine, compared to the past, the efficacy of managerial decisions, supply usage, and weak and strong points for the senior managers and decision-makers. They can improve the efficiency of the units based on this assessment. In this paper, using the data envelopment analysis, the performance of regional electric companies of Iran in 2015 and 2016 is assessed. Because of semi-positive and negative indices, the Slack-Based Measure (SBM) model of efficiency is developed for the 16 regional electric companies of Iran. To determine progress and regression in 2016, compared with 2015, models are proposed to compute the indices of productivity. In the end, solving the proposed models, the Malmquist productivity index is computed for regional electric companies of Iran with 18 input, intermediate, and output indices, and considering the amount of production of renewable energies in one of the important output indices, because of the irrefutable necessity of this kind of energies in the world. Their progress and regression are obtained using the software Gams, showing progress in two companies, and regression in 13 companies, while one company had neither progress nor regression. Studies performed show that agility of the organizational structure, financial and human resource limitations, sanctions, and imbalance between the actual price of production of one Kilowatt-hour of electricity and its sale are factors effective on the progress and regression of the companies.

Keywords: Data envelopment analysis, Slack-based measure of efficacy, Malmquist productivity index, Renewable energy, Semi-positive and negative indices

1. Introduction

In the two recent decades, performance assessment as one of the most important tools to control the country's power industry. Tavanir company was paramount for senior management in downstream companies. To this end, as per the progress of science and technology, manifold models, approaches and tools like KPI, BSC and EFQM are used. The main emphasis of these models is on the achievement of objectives and streaming of processes in the organization, but less on optimizing the spent supplies (efficiency assessment).

The 16 regional electric companies are public, and subordinate to the Tavanir company as an infrastructural and mother company. These companies transmit the electrical energy produced by thermal, hydraulic, atomic, and renewable energy power plants scattered in different regions of Iran to the 16 consuming regions of the country

using the transmission and sub-transmission power networks, voltage conversion, and power-sharing (dispatching). Through the electricity distribution companies, they supply power for their main subscribers in addition to residential, industrial, agricultural, and commercial subscribers. These companies are regional electricity companies of Azerbaijan, Esfahan, Bakhtar, Tehran, Khorasan, Khozestan, Zanjan, Semnan, SistanBalochistan, Gharb, Fars, Kerman, Gilan, Mazandaran, Hormozgan, and Yazd, which are considered in this paper for progress and regression assessment in three stage unit [16]. The power industry of the country (Tavanir) is divided into 16 regions. The model of data envelopment analysis, SBM, and Malmquist are used for assessment. Charnes et al. [1] introduced a non-parametric model for assessing homogeneous Decision-Making Units (DMUs), and called it the

CCR model in Data Envelop Analysis (DEA). Return to scale assumed constant in this model, Banker et. al [2] extended the CCR model by eliminating the condition of efficiency to constant scale, and introduced the BCC model, in which efficiency to variable scale is considered. In these models, positive values of input and output indices is a fundamental hypothesis. Emrouznejad et al. [3] extended the basic models of data envelopment analysis for semi-positive and negative indices. In their method, each negative index is considered as the difference of two positive indices (expense-profit). Before, Emrouznejad et al. [3], Sharp et al. [4] extended the efficiency measure model based on slack variable (SBM) introduced by Tone et al. Since the introduced model is stable against transition, the extended SBM model can be used for negative indices too. Because of the network structure of DEA, intermediate activities intervene in the assessment of units as an intermediate index.

Manthos et al. [6] introduced a new network model using Monte Carlo and Bayesian assessment techniques to measure bank management performance for good management. Since management methods are considered an important lost input in executive processes of banks network, they used management methods as a lost input in the standard production function of banks network. They showed that management has a strong dependence on the bank's profitability and failure likelihood. Puyenbroeck et. al [7] used data envelopment analysis to develop an alternative version of the European Commission's Cultural and Creative Cities Index. They provided an index score, based on network DEA, that reflects expert opinion, cities' diverse socio-economic backgrounds, and observed relative strengths. By identifying and studying three city clusters with highly different performances, they managed to reach an efficient pattern identification. Dickson et. al [8] developed network DEA models for assessing urban water utility efficiency. The introduced model is based on competition criterion. Assuming that the current efficiency of urban water utility should be equal to or better than the previous year's performance, they used the benchmarking method to compute the efficiency of service delivery of urban water utilities in environments without competitors. Finally, they provided a pattern for improving the efficiency. Lozano et. al [9] proposed the smallest improvement DEA approach. They considered various scenarios regarding intermediate indexes. They introduced the improvement pattern so that the relative

distance to the frontier is minimal. The proposed method has been validated on general network DEA too. Zhou et. al [10] used a network DEA model based on dynamic double frontier to evaluate a sustainable supply chain. They used data of the degrees of environmental pollution and customer satisfaction which are type-2 fuzzy data. Their proposed model, in the optimistic attitude toward the decision makers, has high discriminating power and the ability to provide more managerial insights. Moreover, extensive studies are performed on two stage network and SBM model. For example, Zhou et al [11] studied the wastewater treatment systems in China and considered it as two stages. Firstly, the classical SBM model is computed for the first and second stages of each DMU. Then, the objective function for the SBM model of the total network is considered as the convex combination of the first and second stage objective functions and the constraints of the overall network model are set as the sum of the constraints of the SBM model for the first and second stages. Furthermore, the wastewater treatment network is considered as a black box and the SBM model is written. Their method is inspired in this paper and developing the model of Sharp et al., the network SBM model is obtained to compute the progress and regression of regional electric companies. Inperformance assessment, the progress, and regression of a decision-making unit for improving efficiency and productivity was first broached by the idea of dividing the current efficiency to the past one. Because of various difficulties, Malmquist productivity index was introduced by Malmquist [12] in 1953, which shows progress and regression of decision-making units. Malmquist productivity index is based on the data envelop analysis model, and based on the distance function of the technical efficiency of the observations of the unit 0 at time t and $t + 1$ in relation to production boundaries with efficiency technology to a constant scale of time t and $t + 1$ [17]. The aim of Ding et al. [13] is to study dynamic evolution of Industrial Circular Economy (ICE). Using network data envelop analysis, they measured the overall efficiency, sub-system efficiency, and factor efficiency of the ICE system. Then the Malmquist index is extended for assessing cooperative game network. Their evaluation shows that the overall efficiency of dynamic evolution industrial circular economy system has stark contrasts among cities and sub-systems. Pastor et al. [14] introduced a new distance function to introduce a new Malmquist productivity index. The proposed index shows

that productivity change as the ratio of two components; productivity change due to output change in the numerator and productivity change due to input change in the denominator. Liu et al. [15] used the Malmquist productivity index to compute green productivity growth and competition analysis of road transportation at the provincial level. They decomposed green productivity growth into changes in various types of efficiency and technological progress. Decline of the green productivity in China's Eastern area shows there are opportunities for improvement in technology efficiency and foreign transportation efficiency.

The objective of this research is to determine the progress and regression of 16 regional electric companies of Iran in the years 2015 and 2016. To this end, the existing network models are extended for Iran's regional electric network. Section 2 is a literature review including extended network NSBM for assessing the progress and regression of units. Section 3 explains the research methodology including extended network SBM (NSMB) for the network of Iran's regional electric companies and the Malmquist productivity index for Iran's regional electric network. Section 4 is a case study for computing the Malmquist productivity index by extending the SBM model for Iran's 16 regional electric companies. Section 5 is the conclusion and suggestions for progress and regression of the 16 regional electric companies of Iran in the years 2015 and 2016.

2. Extended network NSBM model for assessment of unit progress and regression

Assume n DMU_j, ($j = 1, \dots, n$) units are under assessment. $X_j = (x_{1j}, \dots, x_{mj})$ is consumption input, and $Y_j = (y_{1j}, \dots, y_{sj})$ is the output of DMU_j. It is necessary to describe the SBM model by strictly positive indices and semi-positive and negative indices.

2.1. SBM model to measurement efficiency based on slack variable

The SBC efficiency measurement model, which is based on the slack variable and is introduced by Tone [5], is directly related to scale measurement with input process and output deficiencies of the Decision-Making Unit (DMU). The SBM model introduced by Tone et al. is described by strictly positive inputs and outputs based on model (1).

$$\text{Min } \rho = \frac{1 - \left(\frac{1}{m}\right) \sum_{i=1}^m \frac{s_i^-}{x_{i0}}}{1 + \left(\frac{1}{s}\right) \sum_{r=1}^s \frac{s_r^+}{y_{r0}}} \quad (1)$$

s.t.

$$\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{i0}; i = 1, \dots, m$$

$$\sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{r0}; r = 1, \dots, s$$

$$\sum_{j=1}^n \lambda_j = 1$$

$$s_i^- \geq 0; s_r^+ \geq 0; i = 1, \dots, m; r = 1, \dots, s$$

$$\lambda_j \geq 0; j = 1, \dots, n.$$

Assume that the optimum solution of model (1) in the assessment of DMU₀ is $\rho^* = 1$. Then DMU₀ is called SBM efficient.

2.2. SBM model extended to measuring efficiency with semi-positive and negative indices

Sharp et al. [4] extended the model introduced by Tone. [5] as BB for semi-positive and negative indices by combining a weight set for inputs and outputs.

The extended model of SBM, i.e. NSBM is in the form of model (2).

$$\rho^* = \text{Min } \rho = \frac{1 - \sum_{i=1}^m \frac{w_i s_i^-}{R_{i0}}}{1 + \sum_{r=1}^s \frac{v_r s_r^+}{R_{r0}}} \quad (2)$$

s.t.

$$\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{i0}; i = 1, \dots, m$$

$$\sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{r0}; r = 1, \dots, s$$

$$\sum_{j=1}^n \lambda_j = 1$$

$$s_i^- \geq 0; s_r^+ \geq 0; i = 1, \dots, m; r = 1, \dots, s$$

$$w_i \geq 0; v_r \geq 0; i = 1, \dots, m; r = 1, \dots, s$$

$$\lambda_j \geq 0; j = 1, \dots, n$$

where $\sum_{r=1}^s v_r = 1$ and $\sum_{i=1}^m w_i = 1$. R_{i0} and R_{r0} are defined as

$$R_{i0} = x_{i0} - \min_j \{x_{ij}\}; i = 1, \dots, m$$

$$R_{r0} = \max_j \{y_{rj}\} - y_{r0}; r = 1, \dots, s \quad (3)$$

v_r and w_i are determined by the Decision-Maker (DM). If $v_r = \frac{1}{s}$ ($r = 1, \dots, s$) and $w_i = \frac{1}{m}$ ($i = 1, \dots, m$), the objective function can be substituted by Equation (4), which is the final Sharp model.

$$\rho = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i^-}{R_{i0}}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{R_{r0}}} \quad (4)$$

Assuming that $\rho^* = 1$ is the optimum solution of the model (2) in the assessment of DMU₀, then

DMU₀ is called SBM efficient. Otherwise, it is called SBM inefficient.

3. Research methodology

In this section, we describe the network SBM model (three stage) for performance assessment of the 16 regional electric companies of Iran. Firstly, we describe the general structure of the network of regional electric companies, and then propose a model for evaluating them. In the end, the proposed model is extended to determine the progress and regression of the companies for two successive years.

3.1. Extended network SBM model for network of Iran's regional electric companies

Assume that n DMU _{j} ($j = 1, \dots, n$) are under assessment. Activities of each unit with inputs and outputs as the indices are shown in Conceptual (Figure 1 in three stages of administrative and financial, design, and development, and operation along with their network connections (named intermediate indices). Functions of the administrative and financial stage are: recruitment, education, the welfare of the human resource, planning, oversight, adapting budget with productivity, resource providing and accounting. Functions of design and development are: installation and development of transmission and sub-transmission installations as well as the construction of Distribution Generation (DG) and renewable energy power plants. Functions of the operation stage are: operation and maintenance of the installations and equipment.

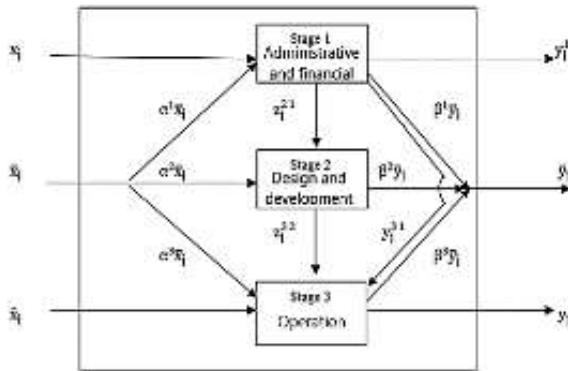


Figure 1. Conceptual model and structure of Iran's regional electric companies.

According to figure 1, we have:

$X_j = (x_{1j}, \dots, x_{m1j})$: input of the first stage of DMU _{j}

$\bar{X}_j = (\bar{x}_{1j}, \dots, \bar{x}_{m2j})$: the input devoted to the first, second, and third stages. The ratio devoted to each stage is shown by $\alpha^1, \alpha^2, \alpha^3$ where $\sum_{t=1}^3 \alpha^t = 1$.

$Y_j^1 = (y_{1j}^1, \dots, y_{s1j}^1)$: output of DMU _{j} first stage, which exists the system.

$Z^{12}_j = (z_{1j}^{12}, \dots, z_{t1j}^{12})$: intermediate index between the first and second DMU _{j} (namely the output of the first stage of DMU _{j} , which is the input of the second stage).

$Z^{23}_j = (z_{1j}^{23}, \dots, z_{t2j}^{23})$: intermediate index between the second and third DMU _{i} (namely the output of the second stage of DMU _{i} which is the input of the third stage).

$\hat{X}_j = (\hat{x}_{1j}, \dots, \hat{x}_{m3j})$: the input index of stage (3) of DMU _{j}

$Y^{13}_j = (y_{1j}^{13}, \dots, y_{s3j}^{13})$: intermediate index between the first and third DMU _{j} (namely the output of the first stage of DMU _{j} , which is the input of the third stage).

$\bar{Y}_j = (\bar{y}_{1j}, \dots, \bar{y}_{s3j})$: the common output by the first, second, and third stages. The ratio devoted to each stage is shown by $\beta^1, \beta^2, \beta^3$, where $\sum_{t=1}^3 \beta^t = 1$.

$Y_j = (y_{1j}, \dots, y_{s4j})$: the output produced by DMU _{j} in the third stage.

The conceptual form corresponds to the activities performed in the regional electric companies of Iran. The three stages, their indices, and the conceptual structure are further discussed in section 4.

In this section, we develop the SBM model in section 2, developed for the regional electric network of Iran considering the structure of the conceptual figure 1.

Assume that DMU _{p} is the unit under assessment. One has

$$\begin{aligned} \bar{\rho}^* &= \text{Min } \bar{\rho} = \\ & 1 - \frac{1}{m_1 + 3m_2 + m_3} \left(\sum_{i=1}^{m_1} \frac{s_i^{(1)-}}{R_i^1} + \sum_{i=1}^{m_2} \frac{s_i^{(2)-}}{R_i^2} + \sum_{i=1}^{m_2} \frac{s_i^{(3)-}}{R_i^3} + \sum_{i=1}^{m_2} \frac{s_i^{(4)-}}{R_i^4} + \sum_{i=1}^{m_3} \frac{s_i^{(5)-}}{R_i^5} \right) \\ & 1 + \frac{1}{s_1 + s_3 + s_5} \left(\sum_{r=1}^{s_1} \frac{s_r^{(1)+}}{R_r^1} + \sum_{r=1}^{s_3} \frac{s_r^{(2)+}}{R_r^2} + \sum_{r=1}^{s_3} \frac{s_r^{(3)+}}{R_r^3} + \sum_{r=1}^{s_3} \frac{s_r^{(4)+}}{R_r^4} + \sum_{r=1}^{s_4} \frac{s_r^{(5)+}}{R_r^5} \right) \\ & \text{s.t.} \\ & \sum_{j=1}^n \lambda_j^1 x_{ij} + s_i^{(1)-} = x_{ip} ; i = 1, \dots, m_1 \\ & \sum_{j=1}^n \lambda_j^1 (\alpha^1 \bar{x}_{ij}) + s_i^{(2)-} = \alpha^1 \bar{x}_{ip} ; i = 1, \dots, m_1 \\ & \sum_{j=1}^n \lambda_j^1 y_{rj}^1 - s_r^{(1)+} = y_{rp}^1 ; r = 1, \dots, s_1 \\ & \sum_{j=1}^n \lambda_j^1 (\beta^1 \bar{y}_{rj}) - s_r^{(2)+} = \beta^1 \bar{y}_{rp} ; r = 1, \dots, s_r \\ & \sum_{j=1}^n \lambda_j^1 z_{tj}^{12} \geq \sum_{j=1}^n \lambda_j^2 z_{tj}^{12} ; t = 1, \dots, t_1 \\ & \sum_{j=1}^n \lambda_j^1 y_{rj}^{13} \geq \sum_{j=1}^n \lambda_j^3 y_{rj}^{13} ; r = 1, \dots, s_2 \\ & \sum_{j=1}^n \lambda_j^2 (\alpha^2 \bar{x}_{ij}) + s_i^{(3)-} = \alpha^2 \bar{x}_{ip} ; i = 1, \dots, m_2 \\ & \sum_{j=1}^n \lambda_j^2 (\beta^2 \bar{y}_{rj}) - s_r^{(3)+} = \beta^2 \bar{y}_{rp} ; r = 1, \dots, s_3 \\ & \sum_{j=1}^n \lambda_j^2 z_{tj}^{23} \geq \sum_{j=1}^n \lambda_j^3 z_{tj}^{23} ; t = 1, \dots, t_2 \\ & \sum_{j=1}^n \lambda_j^3 (\alpha^3 \bar{x}_{ij}) + s_i^{(4)-} = \alpha^3 \bar{x}_{ip} ; i = 1, \dots, m_2 \\ & \sum_{j=1}^n \lambda_j^3 (\beta^3 \bar{y}_{rj}) - s_r^{(4)+} = \beta^3 \bar{y}_{rp} ; r = 1, \dots, s_3 \end{aligned} \quad (5)$$

$$\begin{aligned} \sum_{j=1}^n \lambda_j^3 y_{rj} - s_r^{(5)+} &= y_{rp}; r = 1, \dots, s_4 \\ \sum_{j=1}^n \lambda_j^1 &= 1, \quad \sum_{j=1}^n \lambda_j^2 = 1, \quad \sum_{j=1}^n \lambda_j^3 = 1 \\ \Lambda_j^1 &\geq 0, \Lambda_j^2 \geq 0, \Lambda_j^3 \geq 0; j=1, \dots, N \end{aligned}$$

where

$$\begin{aligned} R_i^1 &= x_{i0} - \min_j \{x_{ij}\}; i = 1, \dots, m_1 \\ R_i^3 &= (\alpha^2 \bar{x}_{i0}) - \min_j \{\alpha^2 \bar{x}_{ij}\}; i = 1, \dots, m_2 \\ R_i^4 &= (\alpha^3 \bar{x}_{i0}) - \min_j \{\alpha^3 \bar{x}_{ij}\}; i = 1, \dots, m_2 \\ R_i^5 &= \bar{x}_{i0} - \min_j \{\bar{x}_{ij}\}; i = 1, \dots, m_3 \\ R_r^1 &= \max_j \{y_{rj}^1\} - (y_{ro}^1); r = 1, \dots, s_1 \\ R_r^2 &= \max_j \{\beta^1 \bar{y}_{ro}\} - (\beta^1 \bar{y}_{ro}); r = 1, \dots, s_3 \\ R_r^3 &= \max_j \{\beta^2 \bar{y}_{rj}\} - (\beta^2 \bar{y}_{ro}); r = 1, \dots, s_3 \\ R_r^4 &= \max_j \{\beta^3 \bar{y}_{rj}\} - (\beta^3 \bar{y}_{ro}); r = 1, \dots, s_3 \\ R_R^5 &= \max_j \{y_{Rj}\} - (y_{Ro}); R = 1, \dots, s_4 \end{aligned} \quad (6)$$

Definition 1. Assume that in the assessment of DMU_p we have $\bar{\rho}^* = 1$ ($\bar{\rho}^*$ is the optimum of model 5). Then DMU_p is called NSBM efficient (network SBM). Otherwise, called DMU_p, an inefficient NSB.

We are going to use the optimum solution of the model (5) for computing the efficient of every stage. Assume

$$\left(s^{(1)-*}, s^{(2)-*}, s^{(3)-*}, s^{(4)-*}, s^{(1)+*}, s^{(2)+*}, s^{(3)+*}, s^{(4)+*} \right), s^{(5)+*}, \lambda^1, \lambda^2, \lambda^3, \bar{\rho}^*$$

is the optimum solution of the model (5) in the assessment of DMU_p. Let

$$\bar{p}^{*(1)} = \frac{1 - \frac{1}{M_1 + M_2} \left(\sum_{i=1}^{M_1} \frac{s_i^{(1)-*}}{R_i^1} + \sum_{i=1}^{M_2} \frac{s_i^{(2)-*}}{R_i^2} \right)}{1 + \frac{1}{s_1 + s_3} \left(\sum_{r=1}^{s_1} \frac{s_r^{(1)+*}}{R_r^1} + \sum_{r=1}^{s_3} \frac{s_r^{(2)+*}}{R_r^2} \right)} \quad (7)$$

$$\bar{p}^{*(2)} = \frac{1 - \frac{1}{M_2} \left(\sum_{i=1}^{M_2} \frac{s_i^{(3)-*}}{R_i^3} \right)}{1 + \frac{1}{s_3} \left(\sum_{r=1}^{s_3} \frac{s_r^{(3)+*}}{R_r^3} \right)} \quad (8)$$

$$\bar{p}^{*(3)} = \frac{1 - \frac{1}{M_2 + M_3} \left(\sum_{i=1}^{M_2} \frac{s_i^{(4)-*}}{R_i^4} + \sum_{i=1}^{M_3} \frac{s_i^{(5)-*}}{R_i^5} \right)}{1 + \frac{1}{s_3 + s_4} \left(\sum_{r=1}^{s_3} \frac{s_r^{(4)+*}}{R_r^4} + \sum_{r=1}^{s_4} \frac{s_r^{(5)+*}}{R_r^5} \right)} \quad (9)$$

$\bar{p}^{*(1)}$, $\bar{p}^{*(2)}$ and $\bar{p}^{*(3)}$ show the efficacy of each stage of DMU_p. It is evident that if the efficacy of the first, second, and third stages are equal to one; then

$$\begin{aligned} s_j^{(3)-*} = s_r^{(3)+*} &= 0^0, \quad s_r^{(2)+*} = s_r^{(1)+*} = s_j^{(1)-*} = s_j^{(2)-*} = \\ 0^0, \quad s_j^{(4)-*} &= s_j^{(5)-*} = s_r^{(4)+*} = s_r^{(5)+*} = 0^0 \end{aligned}$$

In this case, considering the objective function of the model (5), the overall efficacy will be equal to one too. Moreover, if DMU_p is totally efficient it can be deduced that all slack variables of input, output, and intermediate indexes defined in the objective function of the model (5) are equal to zero, which results that the efficacies of all subunits are equal to one. Thus if DMU_p is totally efficient, then all of its subunits are efficient, and vice versa. To linearize the model (5) we use the Charnes Cooper transform (1987). Let

$$\begin{aligned} 1 + \frac{1}{s_1 + 3s_3 + s_5} \left(\sum_{r=1}^{s_1} \frac{s_r^{(1)+}}{R_r^1} + \sum_{r=1}^3 \frac{s_r^{(2)+}}{R_r^2} + \sum_{r=1}^{s_3} \frac{s_r^{(3)+}}{R_r^3} + \right. \\ \left. \sum_{r=1}^{s_3} \frac{s_r^{(4)+}}{R_r^4} + \sum_{r=1}^{s_4} \frac{s_r^{(5)+}}{R_r^5} \right) = \frac{1}{h}, \quad \Lambda_j^1 = h\Lambda_j^1, \quad s_i^{(1)-} = h s_i^{(1)-}, \\ s_i^{(2)-} = h s_i^{(2)-}, \quad s_r^{(1)+} = h s_r^{(1)+}, \quad s_r^{(2)+} = h s_r^{(2)+}, \\ \Lambda_j^2 = h\Lambda_j^2, \quad s_i^{(3)-} = h s_i^{(3)-}, \quad s_r^{(3)+} = h s_r^{(3)+}, \quad \Lambda_j^3 = h\Lambda_j^3, \\ s_r^{(4)+} = h s_r^{(4)+}, \quad s_r^{(5)+} = h s_r^{(5)+} \end{aligned}$$

In this case, the rational model (5) transforms to the linear model (10), as follows.

$$\begin{aligned} \hat{\rho}^* = \text{Min } \hat{\rho} = h - \frac{1}{m_1 + 3m_2 + m_3} \left(\sum_{i=1}^{m_1} \frac{s_i^{(1)-}}{R_i^1} + \right. \\ \left. \sum_{i=1}^{m_2} \frac{s_i^{(2)-}}{R_i^2} + \sum_{i=1}^{m_2} \frac{s_i^{(3)-}}{R_i^3} + \sum_{i=1}^{m_2} \frac{s_i^{(4)-}}{R_i^4} + \right. \\ \left. \sum_{i=1}^{m_3} \frac{s_i^{(5)-}}{R_i^5} \right) \end{aligned} \quad (10)$$

s.t.

$$\begin{aligned} \sum_{j=1}^n \lambda_j^1 x_{ij} + s_i^{(1)-} &= h x_{ip}; i = 1, \dots, m_1 \\ \sum_{j=1}^n \lambda_j^1 (\alpha^1 \bar{x}_{ij}) + s_i^{(2)-} &= h (\alpha^1 \bar{x}_{ip}); i = 1, \dots, m_2 \\ \sum_{j=1}^n \lambda_j^1 y_{rj} - s_r^{(1)+} &= h y_{rp}; r = 1, \dots, s_1 \\ \sum_{j=1}^n \lambda_j^1 (\beta^1 \bar{y}_{rj}) - s_r^{(2)+} &= h (\beta^1 \bar{y}_{rp}); r = 1, \dots, s_3 \\ \sum_{j=1}^n \lambda_j^1 z_{tj}^1 &\geq \sum_{j=1}^n \lambda_j^2 z_{tj}^1; t = 1, \dots, t_1 \\ \sum_{j=1}^n \lambda_j^1 y_{rj}^1 &\geq \sum_{j=1}^n \lambda_j^3 y_{rj}^1; r = 1, \dots, s_2 \\ \sum_{j=1}^n \lambda_j^2 (\alpha^2 \bar{x}_{ij}) + s_i^{(3)-} &= h (\alpha^2 \bar{x}_{ip}); i = 1, \dots, m_2 \\ \sum_{j=1}^n \lambda_j^2 (\beta^2 \bar{y}_{rj}) - s_r^{(3)+} &= h (\beta^2 \bar{y}_{rp}); r = 1, \dots, s_3 \\ \sum_{j=1}^n \lambda_j^2 z_{tj}^2 &\geq \sum_{j=1}^n \lambda_j^3 z_{tj}^2; t = 1, \dots, t_2 \\ \sum_{j=1}^n \lambda_j^3 (\alpha^3 \bar{x}_{ij}) + s_i^{(4)-} &= h (\alpha^3 \bar{x}_{ip}); i = 1, \dots, m_2 \\ \sum_{j=1}^n \lambda_j^3 (\beta^3 \bar{y}_{rj}) - s_r^{(4)+} &= h (\beta^3 \bar{y}_{rp}); r = 1, \dots, s_3 \\ \sum_{j=1}^n \lambda_j^3 y_{rj} - s_r^{(5)+} &= h (y_{rp}); r = 1, \dots, s_4 \\ h + \frac{1}{s_1 + 3s_3 + s_5} \left(\sum_{r=1}^{s_1} \frac{s_r^{(1)+}}{R_r^1} + \sum_{r=1}^{s_3} \frac{s_r^{(2)+}}{R_r^2} + \sum_{r=1}^{s_3} \frac{s_r^{(3)+}}{R_r^3} + \right. \\ \left. \sum_{r=1}^{s_3} \frac{s_r^{(4)+}}{R_r^4} + \sum_{r=1}^{s_4} \frac{s_r^{(5)+}}{R_r^5} \right) &= 1 \\ \sum_{j=1}^n \lambda_j^1 &= 1, \quad \sum_{j=1}^n \lambda_j^2 = 1, \quad \sum_{j=1}^n \lambda_j^3 = 1 \\ \lambda_j^1 &\geq 0, \lambda_j^2 \geq 0, \lambda_j^3 \geq 0; j=1, \dots, n. \end{aligned}$$

Definition 2. Assume that the optimum solution of the model (10) in assessment DMU_p is $\bar{p}^* = 1$. Then DMU_p is called NSBM efficient (network SBM). Otherwise, DMU_p is called NSBM inefficient.

Evidently, the model (10) is invariant to translation. Thus the units with semi-positive and negative indices are also assessed by the model (10).

3.2. Malmquist productivity index i regional electric network of Iran

Assume n unit under assessment at time $t + 1$ and $T = t$, DMU_j^T ($j = 1, \dots, n$) are given and each one has a structure similar to figure 1. In this section, we are going to develop the network SBM model introduced in section 3 to be able to assess the progress and regression of DMU_p at time $t + 1$ compared to the time t . Consider the following model.

$$\theta^* K(X_p^L, Y_p^L) = \text{Min } h - \frac{1}{m_1 + 3m_2} \left(\sum_{i=1}^m \frac{s_i^{(1)-}}{R_i^1} + \sum_{i=1}^m \frac{s_i^{(2)-}}{R_i^2} + \sum_{i=1}^m \frac{s_i^{(3)-}}{R_i^3} + \sum_{i=1}^m \frac{s_i^{(4)-}}{R_i^4} + \sum_{i=1}^m \frac{s_i^{(5)-}}{R_i^5} \right) \quad (11)$$

$$\begin{aligned} \text{s.t.} \\ \sum_{j=1}^n \lambda_j^1 x_{ij}^K + s_i^{(1)-} &= (h - \lambda_p^1) x_{ip}^L; i = 1, \dots, m_1 \\ \sum_{j=1}^n \lambda_j^1 (\alpha^1 \bar{x}_{ij}^K) + s_i^{(2)-} &= (h - \lambda_p^1) (\alpha^1 \bar{x}_{ip}^L); i = 1, \dots, m_2 \\ \sum_{j=1}^n \lambda_j^1 y_{rj}^{1K} - s_r^{(1)+} &= (h - \lambda_p^1) \bar{y}_{rp}^L; r = 1, \dots, s_1 \\ \sum_{j=1}^n \lambda_j^1 (\beta^1 \bar{y}_{rj}^K) - s_r^{(2)+} &= (h - \lambda_p^1) (\beta^1 \bar{y}_{rp}^L); r = 1, \dots, s_3 \\ \sum_{j=1}^n \lambda_j^1 z_{tj}^{21k} &\geq \sum_{j=1}^n \lambda_j^2 z_{tj}^{12L}; t = 1, \dots, t_1 \\ \sum_{j=1}^n \lambda_j^1 y_{rj}^{13k} &\geq \sum_{j=1}^n \lambda_j^3 y_{rj}^{13L}; r = 1, \dots, s_2 \\ \sum_{j=1}^n \lambda_j^2 (\alpha^2 \bar{x}_{ij}^K) + s_i^{(3)-} &= (h - \lambda_p^2) (\alpha^2 \bar{x}_{ip}^L); i = 1, \dots, m_2 \\ \sum_{j=1}^n \lambda_j^2 (\beta^2 \bar{y}_{rj}^K) - s_r^{(3)+} &= (h - \lambda_p^2) (\beta^2 \bar{y}_{rp}^L); r = 1, \dots, s_3 \end{aligned}$$

$$\begin{aligned} \sum_{j=1}^n \lambda_j^2 z_{tj}^{32k} &\geq \sum_{j=1}^n \lambda_j^3 z_{tj}^{32L}; t = 1, \dots, t_2 \\ \sum_{j=1}^n \lambda_j^3 (\alpha^3 \bar{x}_{ij}^K) + s_i^{(4)-} &= (h - \lambda_p^3) (\alpha^3 \bar{x}_{ip}^L); i = 1, \dots, m_3 \\ \sum_{j=1}^n \lambda_j^3 \bar{x}_{ij}^K + s_i^{(5)-} &= (h - \lambda_p^3) \bar{x}_{ip}^L; i = 1, \dots, m_3 \\ \sum_{j=1}^n \lambda_j^3 (\beta^3 \bar{y}_{rj}^K) - s_r^{(4)+} &= (h - \lambda_p^3) (\beta^3 \bar{y}_{rp}^L); r = 1, \dots, s_3 \\ \sum_{j=1}^n \lambda_j^3 y_{rj}^K - s_r^{(5)+} &= (h - \lambda_p^3) \bar{y}_{rp}^L; r = 1, \dots, s \\ h + \frac{1}{s_1 + 3s_3 + s_5} \left(\sum_{r=1}^{s_1} \frac{s_r^{(1)+}}{R_r^1} + \sum_{r=1}^{s_3} \frac{s_r^{(2)+}}{R_r^2} + \sum_{r=1}^{s_3} \frac{s_r^{(3)+}}{R_r^3} + \sum_{r=1}^{s_3} \frac{s_r^{(4)+}}{R_r^4} + \sum_{r=1}^{s_4} \frac{s_r^{(5)+}}{R_r^5} \right) &= 1 \\ \sum_{j=1}^n \lambda_j^1 &= h, \quad \sum_{j=1}^n \lambda_j^2 = h, \quad \sum_{j=1}^n \lambda_j^3 = h \\ \lambda_j^1 \geq 0, \lambda_j^2 \geq 0, \lambda_j^3 \geq 0; j=1, \dots, N \end{aligned}$$

If in the model (11) one uses

- i₁) $K = t$ and $L = t$
- i₂) $K = t$ and $L = t + 1$
- i₃) $K = t + 1$ and $L = t$
- i₄) $K = t + 1$ and $L = t + 1$

then four new models are made. We call the optimum solution of each one $\theta^{*t}(x_p^t, y_p^t)$, $\theta^{*t}(x_p^{t+1}, y_p^{t+1})$, $\theta^{*t+1}(x_p^t, y_p^t)$, and $\theta^{*t+1}(x_p^{t+1}, y_p^{t+1})$, respectively.

Note that the PPS of each model is different. For example, in PPS of the model (6) the unit under assessment DMU_p is at time L (i.e. DMU_p^L = (X_p^L, Y_p^L)). and other units are at time L (i.e. DMU_j^K = (X_j^K, Y_j^K), $j = 1, \dots, n, j \neq p$). If we show the PPS of this model by PPS^K(X₀^L, X₀^L) then the PPS of the models 3, 4, 5 and 6 are called PPS^t(x_p^t, y_p^t), PPS^t(x_p^{t+1}, y_p^{t+1}), PPS^{t+1}(x_p^t, y_p^t) and PPS^{t+1}(x_p^{t+1}, y_p^{t+1}) respectively.

Therefor

$R_i^1, R_i^2, \dots, R_i^s$ will depend on their PPS in (s) models. for example, for the model (11) one has

$$\begin{aligned} R_i^1 &= (x_{ip}^L) - \min_{j \in J_L^K} \{x_{ij}\}; i = 1, \dots, m_1 \\ R_i^2 &= (\alpha^1 \bar{x}_{ip}^L) - \min_{j \in J_L^K} \{\alpha^1 \bar{x}_{ij}\}; i = 1, \dots, m_2 \\ R_i^3 &= (\alpha^2 \bar{x}_{io}^L) - \min_{j \in J_L^K} \{\alpha^2 \bar{x}_{ij}\}; i = 1, \dots, m_2 \\ R_i^4 &= (\alpha^3 \bar{x}_{io}^L) - \min_{j \in J_L^K} \{\alpha^3 \bar{x}_{ij}\}; i = 1, \dots, m_2 \\ R_r^1 &= \max_{j \in J_L^K} \{y_{rj}^1\} - (y_{ro}^L); r = 1, \dots, s_1 \\ R_r^2 &= \max_{j \in J_L^K} \{\beta^1 \bar{y}_{rj}\} - (\beta^1 \bar{y}_{ro}^L); r = 1, \dots, s_3 \end{aligned}$$

$$R_r^3 = \max_{j \in J_L^L} \{\beta^2 \bar{y}_{rj}\} - (\beta^2 \bar{y}_{r0}^L); r = 1, \dots, s_3$$

$$R_r^4 = \max_{j \in J_L^K} \{\beta^3 \bar{y}_{rj}\} - (\beta^3 \bar{y}_{r0}^L); r = 1, \dots, s_3$$

$$R_R^5 = \max_{j \in J_L^K} \{Y_{Rj}\} - (Y_{R0}^L); R = 1, \dots, s_4$$

where, J_L^K is the set of DMUs observed in $PPS^K(X_p^L, X_p^L)$.

Malmquist productivity index for DMU_p is computed using the following relation.

$$MPI_p = \left[\frac{\theta^{*t}(x_p^{t+1}, y_p^{t+1})}{\theta^{*t}(x_p^t, y_p^t)} \cdot \frac{\theta^{*t+1}(x_p^{t+1}, y_p^{t+1})}{\theta^{*t+1}(x_p^t, y_p^t)} \right]^{\frac{1}{2}}$$

$$= \frac{\theta^{*t+1}(x_p^{t+1}, y_p^{t+1})}{\theta^{*t}(x_p^t, y_p^t)} \cdot \left[\frac{\theta^{*t}(x_p^t, y_p^t)}{\theta^{*t+1}(x_p^t, y_p^t)} \cdot \frac{\theta^{*t}(x_p^{t+1}, y_p^{t+1})}{\theta^{*t+1}(x_p^{t+1}, y_p^{t+1})} \right]^{\frac{1}{2}} \quad (12)$$

= (efficiency changes) $\times$ (technical changes)

If $MPI_p > 1$, then DMU_p has progress at time $t + 1$, compared to the time t . If $MPI_p < 1$, then DMU_p has regression at time $t + 1$, compared to the time t , and if $MPI_p = 1$, then DMU_p has not changed at time $t + 1$, compared to the time t (no progress no regression).

4. Case study of computing Malmquist productivity index by developing SBM model: 16 regional electric companies of Iran

In this section, the progress and regression of 16 regional electric companies of Iran in the years 2015 and 2016 are computed using the model in section 3.1. 18 input, output and intermediate indexes in three stages of administrative financial, design and development, and operation in the conceptual Figure (1) are as follows.

Five input indices including:

(x_{1j}); Human resource (people), human resource recruited permanently by the company.

(x_{2j}); Companies record (year), record of the company from the formation to the year 2016.

($\bar{x}_{1j}$); Financial resources (revenue and other resources) (million Rials), the financial resources earmarked to the company by the Tavanir company, and the revenue from selling electricity, subscription charge, and other revenues.

($\hat{x}_{1j}$); Transmission and sub-transmission lines up to 2015 (Kilometer circuit), transmission lines including 400 and 230 KV lines and sub-transmission lines including 132 and 63 KV lines that transmit the energy from one point to another point in the country.

($\hat{x}_{2j}$); Capacity of transmission and sub-transmission sub-stations up to 2015 (MVA), transmission substations include 400 and 230 KV sub-stations and sub-transmission substations

include 132 and 63 KV sub-stations. Their function is to convert the electric potential (voltage) from high voltage to low voltage and vice versa. Distribution of electrical energy to different consuming points of the electric network is also up to them.

Four intermediate indices including:

($z_{1j}^{1,2}$); Human resource for design and development (people), the human resources permanently recruited in the executive design and development unit.

($y_{1j}^{1,3}$); Human resource for operation (people), the human resources permanently recruited in the operational unit.

($z_{1j}^{2,3}$); Transmission and sub-transmission energy lines constructed in 2016 (Kilometer circuit).

($z_{2j}^{2,3}$); Transmission and sub-transmission sub-stations constructed in 2016 (MVA).

Nine output indices:

(y_{1j}^1); Gross profit (loss) (million Rials), sell of electricity minus its actual cost.

(y_{2j}^1); Other net non-operational revenues (costs) (million Rials), selling surplus properties, non-operational revenue (other than electricity) minus related costs.

(y_{3j}^1); Profit (loss) (million Rials), gross loss minus general administrative costs, cost of sales, and financial expenses.

(y_{4j}^1); Activities for the establishment of manager provision plan, customer respect, organizational excellence (qualitative), the existence of a system for manager training, attitude, and respect for stakeholders in and out of the company, and existence of the software for the system of organizational excellence.

($\bar{y}_{1j}$); Attraction of expense and consumptions from the plan (percentage), the amount of expenditure as per the annual consumption plan of the company, maintenance, ... as well as the expense in the execution of investment projects of the company.

(y_{1j}); Buying energy from IGMC and private section (million Kilowatt-hours), buying electrical energy from the electricity market (Iran grid management company) or private section.

(y_{2j}); Selling energy to IGMC and subscribers (million Kilowatt-hours), selling electrical energy including renewable energies to the electricity market (Iran grid management company) or to subscribers having direct contracts.

(y_{3j}); Energy loss in transmission and sub-transmission networks (percent), the amount of electrical energy that is squandered in the form of

heat in transmission lines, transformers and other electrical equipment.

(y_{4j}); Non-distributed energy, the amount of electrical energy that due to contingencies, outage of equipment and installation as planned for maintenance and emergency outage of equipment and installation due to technical defects cause outage for subscribers and is computed as a percentage of the total energy.

Among input and output indices of 2015 and 2016 the indexes y_{2j}^1 , y_{3j}^1 , and y_{1j} are semipositive and negative, the index y_{4j}^1 is qualitative and other indices are strictly positive. Since the managers and policy-makers in most countries have understood that transition from limited fossil energy to renewable energies is not an option nowadays, but an undeniable necessity, the amount of renewable energies is considered in one of the important output indices. According to the designed model, data is fed to the software Gams. The values of the productivity index computed by the relation (11) is shown in table 1. The first column on the right hand shows the Malmquist productivity index MPI_j^p for 2016, compared to 2015.

Table 1. values of Malmquist productivity index for 2016, compared to 2015.

DMU	E_{11}	E_{12}	E_{21}	E_{22}	MPI
1	0.87668	0.9179	0.86664	0.95904	0.07641
2	0.9877	0.91778	0.99259	0.96786	0.95187
3	0.98873	0.91764	0.98287	0.95989	0.94945
4	1	1	1	1	1
5	0.00225	0.97381	0.99273	0.99762	0.9931
6	1	0.95023	1	0.95823	0.95422
7	1	0.93436	1	0.97177	0.95288
8	1	0.03524	1	0.09784	0.05872
9	0.87478	0.00295	1	0.88519	0.0546
10	0.99036	0.89339	0.00241	0.91654	0.91276
11	0.98391	0.97997	0.97699	0.98108	1.00008
12	0.98658	0.00577	0.98119	0.00573	0.00584
13	1	0.00653	1	0.01387	0.00952
14	0.96043	0.98761	0.98736	0.91768	0.07762
15	0.91892	0.90159	1	0.93826	0.95946
16	1	0.03519	1	0.01385	0.02208

As shown, units 1 and 11 have progress, units 2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, and 16 have regression and unit 4 has neither progress nor regression. Considering the managerial relation of the researcher with the 16 regional electric companies, the performed studies show that the results obtained, i.e. the progress and regression of units in 2016 compared with 2015, are next to the real world performance and situation of the units. On average, electric energy consumption in the country is about 5.5 percent. Thus, the network and other installations should be developed

commensurately by investment. Considering financial circumstances, units 1 and 11 had more desirable financial resources at their disposal compared to other units that had regression. The size of the units in terms of human resources, peak of energy consumption and agility of the system has an instrumental role in their progress. One important factor effective on the regression of 13 units is that the actual cost of production of one Kilowatt-hour of electricity is not in balance with its sale due to statutory limitations. The sale cost was about 60 percent of the production cost.

This research will continue by studying the progress and regression of units using five-year special data with limitations in the fifth five-year development plan of the country for each stage and using other DEA models.

6. Conclusion

In this paper, the structure of 16 regional electric companies of Iran is designed in a conceptual form showing companies' activities. In this structure, 5 input indices, 4 intermediate indices, and 9 output indices are used. Moreover, the amount of renewable energies is considered in one of the important output indices because of the transition from fossil energy to renewable energies in today's world. Then we extended the SBM model for the companies' network to be invariant to the transition of indices to resolve the problem of semi-positive and negative indices. Finally, extending Malmquist method models, their progress and regression in 2016, compared to 2015 (the years they were under the management of the author) are computed using the software Gams. The results show that 2 units had progress because of agility, having suitable financial and human resources and good performance, while 13 units had regression, and one unit without progress and regression. Investigating the plans and performance of the companies, and considering the managerial association of the author with the units shows that the results are next to the real performance of the companies. Units 1 and 11, compared to 13 other units that had regression, in addition to acquiring and having better financial resources for developing the network and current expenditures. They were also in a better situation in terms of human resources, peak of electricity consumption, and system agility. We are going to continue the research using five-year special data with limitations in the fifth five-year development plan of the country and compute the progress and regression of three stages and all units using other DEA models.

7. References

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